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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

EDITED BY

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Mount Wilson Solar Observatory of the
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MAY 1918

THE RADIAL VELOCITIES OF 16 SOUTHERN STARS Hugh Lutz 201

THE GENERAL MAGNETIC FIELD OF THE SUN: APPARENT VARIATION OF FIELD-STRENGTH
WITH LEVEL IN THE SOLAR ATMOSPHERE . . . G. E. Hale, F. R. Secor, J. H. Moore, and A. S. Brown 204

ON PARALLAXES AND MOTION OF THE BRIGHTEST GALACTIC NEBULAE STARS BETWEEN
GALACTIC LONGITUDES 90° AND 210°—Continued J. H. Moore 224

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THE ASTROPHYSICAL JOURNAL

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AND ASTRONOMICAL PHYSICS

VOLUME XLVII

MAY 1918

NUMBER 4

THE RADIAL VELOCITIES OF 60 SOUTHERN STARS

By JOSEPH LUNT

At the beginning of the year (1917) a new observing program was commenced, embracing all stars south of the equator of types F, G, K, and M, down to magnitude 5.5 in the *Harvard Revised Photometry* (50) for which radial velocities had not been published by Campbell¹ or Adams² in their lists of 915 and 500 stars, respectively, or previously observed here. Five stars from Campbell's *Second Catalogue of Spectroscopic Binary Stars*³ have been included, as well as 22 others subsequently announced in *Lick Observatory Bulletins* to be variable in velocity.

The 24-inch refractor, in conjunction with the four-prism star spectrograph, was employed, the short camera of 16-inch (40.6 cm) focus being used for the first time. The prisms are of light flint and give spectra of approximately the same linear scale as is given by the two dense prisms and short cameras used with the 60-inch reflector at Mount Wilson.

¹ The Radial Velocities of 915 Stars, *Lick Observatory Bulletin*, No. 229.

² The Radial Velocities of 500 Stars, *Mt. Wilson Contr.*, No. 105.

³ *Lick Observatory Bulletin*, No. 181.

TABLE I

H.R. No.	Star's Name	R.A. 1920	S. Decl. 1920	Mag.	Type	No. of Plates	Epoch 1917 +	Radial Velocity	Solar Motion Correction	Corrected Radial Velocity	Range
3	Piscium†	0 ^h 8 ^m	6° 9'	4.68	K	3	.888	-17.2	-1.2	-18.3	6.7
37	Cetus	0 8	18 23	5.47	K	2	.928	-8.0	-3.7	-11.7	4.2
412	Ceti	1 22	15 1	5.19	K	3	.911	-	-8.4	-31.5	2.3
500	Cetus	1 39	4 5	5.27	K	2	.922	-	-7.9	-41.4	1.0
539	Ceti†	1 48	10 44	3.92	K	2	.922	-	-9.5	-	1.7
904*	Eridani	3 15	22 48	5.05	K	3	.668	-	-15.0	-9.3	2.4
1787	Orionis	5 20	0 58	5.15	K	3	.112	-	-17.2	-	4.4
1836	Doradus	5 25	58 59	5.06	K8	3	.144	-	-10.5	-	3.2
2087	Columbae†	5 53	37 8	5.02	K	2	.147	-	-19.8	-	3.1
2140*	Lepus	6 0	26 17	5.18	G5	3	.123	-	-17.8	-	3.0
2275*	Orion	6 16	2 54	5.18	Ma	3	.137	-	-17.8	-	1.5
2409	Monoceros	6 38	9 5	5.32	Kp	3	.205	-	-18.5	-	3.4
2574*	Can. Maj.	6 50	11 56	4.25	K5	5	.159	-	-18.6	-	2.7
2652	Carinae	6 59	51 18	5.02	Mb	2	.147	-	-18.3	-	2.0
2701	Monocerotis	7 6	4 7	5.02	G5	2	.202	-	-17.3	-	1.0
2934*	Carinae	7 34	52 21	4.92	K5	3	.202	-	-17.6	-	5.2
2939	Puppis	7 37	15 4	5.15	K	2	.350	-	-17.9	-	3.1
3123	Puppis	7 56	23 5	5.22	G5	2	.298	-	-11.7	-	2.8
3220	Puppis	8 10	15 32	5.05	K	2	.324	-	-16.8	-	4.0
3484	Hydrae†	8 43	13 15	4.44	G5	2	.257	-	-9.5	-	0.3
3681*	Hydrae	9 13	6 1	5.40	K5	3	.307	-	-12.5	-	2.0
3705	Antliae†	9 26	35 35	4.64	K2	3	.307	-	-14.6	-	3.8
4159	Carinae†	10 33	57 9	4.54	K5	2	.348	-	-11.9	-	2.7
4499	Centaurus†	11 37	61 39	4.88	G	2	.451	-	-9.6	-	2.6
4523	Centaurus	11 43	40 4	5.04	G	2	.465	-	-7.4	-	1.1
4682	Centauri†	12 15	54 42	4.98	Ma	2	.481	-	-7.5	-	2.6
4699*	Corvus	12 17	13 7	5.36	K	3	.432	-	-16.2	-	3.5
4955	Virginis	13 4	10 18	5.26	K	2	.440	-	-7.4	-	3.5
5005*	Virginis	13 28	5 51	4.83	Mb	3	.494	-	-17.9	-	4.3
5172	Centauri†	13 42	51 2	4.68	K	2	.405	-	-3.1	-	0.2
5301	Virgo	14 7	15 56	5.10	Ma	4	.522	-	-6.0	-	3.9

Star	14 17	58 5	4.89	G		2	.539	+ 10.0	- 3.4	+ 6.6	5.7
<i>Centaurus</i> †	14 31	45 54	5.41	K2		3	.570	+ 55.6	+ 0.2	+ 55.4	0.9
<i>Lupus</i>	14 47	1 58	5.00	K		3	.520	+ 84.9	+ 11.2	+ 90.1	0.6
11 <i>Librae</i>	15 30	9 47	4.83	K		3	.448	+ 11.8	+ 8.8	+ 9.7	2.5
37 <i>Librae</i>	15 30	23 34	5.06	K		3	.610	+ 18.5	+ 8.4	+ 16.5	5.6
42 <i>Librae</i>	16 6	29 12	5.16	G5		3	.632	+ 24.9	+ 8.4	+ 16.5	1.0
<i>Scorpio</i>	16 9	11 38	5.50	G5		3	.599	+ 22.2	+ 13.0	+ 9.2	3.2
<i>Ophiuchus</i>	16 23	7 26	5.45	Ma		2	.641	+ 114.2	+ 14.4	+ 114.2	1.4
H <i>Scorpii</i>	16 31	35 5	4.30	Ma		3	.580	+ 1.3	+ 7.4	+ 6.1	5.5
6196	16 37	17 35	5.04	K		4	.615	+ 27.1	+ 12.4	+ 10.6	3.4
6424	17 13	24 12	5.39	K		4	.666	+ 23.4	+ 11.4	+ 16.0	1.7
<i>Ophiuchi</i>	17 13	24 12	5.39	K		4	.666	+ 23.4	+ 11.4	+ 16.0	1.7
<i>Ophiuchus</i>	17 20	0 59	5.34	G		2	.602	+ 70.7	+ 17.0	+ 53.8	1.4
<i>Sagittarius</i>	17 54	30 14	5.27	K5		2	.602	+ 17.8	+ 9.9	+ 7.9	0.8
<i>Pavonis</i>	18 3	62 1	5.48	G		3	.702	+ 26.3	+ 0.7	+ 25.6	1.3
1 <i>Sagittarii</i>	18 7	23 43	5.13	K		3	.655	+ 4.9	+ 11.8	+ 16.7	2.2
21 <i>Sagittarii</i> †	18 21	20 35	4.96	G5		2	.699	+ 10.3	+ 12.6	+ 2.3	1.9
<i>Coronae Aust.</i>	18 42	40 30	5.28	G		2	.680	+ 20.2	+ 6.5	+ 13.7	3.4
7116*	18 47	22 51	4.96	G5		2	.747	+ 12.0	+ 11.7	+ 0.3	1.9
<i>Pavonis</i>	18 52	60 19	5.14	K		2	.701	+ 177.0	+ 0.3	+ 176.6	0.0
d <i>Sagittarii</i>	19 13	19 6	5.03	K5		3	.696	+ 12.3	+ 12.3	+ 28.7	1.8
f <i>Sagittarii</i>	19 42	19 57	5.06	K		4	.712	+ 20.9	+ 11.3	+ 32.2	0.9
ω <i>Sagittarii</i>	19 51	26 31	4.81	G5		2	.706	+ 14.0	+ 9.3	+ 4.7	1.8
b <i>Sagittarii</i> †	19 52	27 23	4.02	K2		3	.688	+ 17.6	+ 9.0	+ 8.6	0.6
42 <i>Capricorni</i>	21 37	14 24	5.28	K		17	.826	+ 3.0†	+ 7.3	+ 4.3†	48.7
c <i>Capricorni</i>	21 41	9 27	5.28	K		2	.880	+ 3.0	+ 8.1	+ 3.1	2.8
ρ <i>Gruis</i>	22 39	41 50	4.86	K		3	.820	+ 29.0	+ 2.2	+ 26.8	5.1
ψ <i>Aquarii</i>	23 12	9 32	4.46	K		3	.811	+ 22.9	+ 1.9	+ 21.0	1.1
94 <i>Aquarii</i>	23 15	13 54	5.27	K		3	.881	+ 5.6	+ 0.9	+ 6.5	4.7
3 <i>Ceti</i>	24 0	10 57	5.16	K		3	.912	+ 40.5	+ 1.9	+ 42.4	3.5
Mean.....											2.6

* Check stars.
 † Velocity of the center of mass of the system (see *Astrophysical Journal*, 47, 134, 1918).
 ‡ Previously announced as variable in velocity (*Lick Observatory Bulletin*).

§ The solar apex assumed as 18h, +30°, and velocity 20 km per second.

|| Difference between extremes of separate plates.

The linear scale of the spectra from the following iron lines toward the red was:

λ	Δ per mm
4247.6	17.2
4340.6 (H γ)	19.3
4376.1	20.1
4528.8	24.1

The measures were made with the Hartmann spectrocomparator in the region between the foregoing lines, with use of the 45 mm objectives, without extension tubes, and the low-power eyepiece. The magnification employed is 20 diameters, and the elevation scale-reading (W) is 13.

TABLE II
COMPARISON OF RESULTS* FOR 16 STARS

H.R. No.	MAG.	TYPE	NO. OF PLATES	RADIAL VELOCITIES			DIFFERENCES	
				Lunt (Cape) (1)	Campbell (Lick) (2)	Adams (Mt. Wilson) (3)	(1)-(2)	(1)-(3)
				Km	Km	Km	Km	Km
994.....	5.05	K	3	+ 25.1	+ 26.3	-1.2
2140.....	5.18	G ₅	3	+178.9	+183	-4.1
2275.....	5.18	Ma	3	+ 47.4	+48.3	-0.9
2574.....	4.25	K ₅	5	+ 98.7	+ 96.7	+2.0
2934.....	4.92	K ₅	3	+ 62.2	+ 61.1	+1.1
3681.....	5.40	K ₅	3	- 9.0	- 7.3	-1.7
4699.....	5.36	K	3	+ 16.2	+12.5	+3.7
5095.....	4.83	Mb	3	+ 17.9	+ 19.2	-19.1†	-1.3	-1.2
5535.....	5.00	K	3	+ 84.9	+ 83.2	+1.7
6048.....	5.50	G ₅	3	- 22.2	-26.3	+4.1
6128.....	5.45	Ma	2	+ 99.8	+97.1	+2.7
6761.....	5.48	G	3	+ 26.3	+ 31	-4.7
7116.....	4.96	G ₅	2	- 12.0	- 12.0	0.0
7515.....	5.06	K	4	+ 20.9	+ 23‡	+16.5‡	-2.1	+4.4
8311.....	5.28	K	2	- 5.0	- 6.5	+1.5
8841.....	4.46	K	3	- 22.9	- 26.9	-28.4	+4.0	+5.5
Mean							-0.5	+2.0
Excluding 8841, possibly variable.....							-0.9	+1.6

* Adams has compared results for 26 stars with Lick values and obtains, for Lick - Mt. Wilson, F and G types, 14 stars, +1.6; K and M types, 12 stars, +0.4. *Mt. Wilson Contr.*, No. 105, p. 14.

† Sign of velocity taken as plus.

‡ Difference suspected as due to variability by Adams.

Under these conditions the same mirror strips between the prisms can be used as were employed with the spectra obtained

with the long camera. The micrometer screw is of half-millimeter pitch and one division on the head (0.01 rev.), approximately $1/5000$ of an inch, is equivalent to a radial velocity shift of 6.6 km per second in the mean of the 12 settings. The mean range, difference between highest and lowest value of velocity given by separate plates, is 2.6 km per second. Plate 4903 of α Tauri was used as standard plate throughout, the shift being taken as +82.34 km per second. As a check on the results a number of stars for which radial velocities have been published by Campbell or Adams or both were included, and 16 of these have been observed. Table II shows a comparison of the results obtained. The Cape results appear to lie between those of Lick and Mount Wilson.

Table I gives the provisional velocities of 60 stars for which two or more spectra have been measured. Only one of these stars shows distinct evidence of variable radial velocity during the period of observation, viz., 42 Capricorni, and for this a preliminary orbit has been computed (*Astrophysical Journal*, 47, 134, 1918).

Other stars may prove to be variable in velocity when the period of observation is extended, and it is noteworthy that 12 of these stars, marked ‡, are variables previously announced. All the computations were made to the second decimal place in kilometers per second and rounded off to a tenth in the tables.

In addition to those stars already known to have high radial velocities the following may be noted:

H.R. No.	Star's Name	Magnitude	Type	Radial Velocity	Solar Motion Correction	Corrected Radial Velocity
				Km	Km	Km
7127.....	ω Pavonis	5.14	K	+177.0	- 0.3	+176.6
2701.....	20 Monocerotis	5.02	G5	+ 78.8	-17.3	+ 61.6
6516.....	<i>Ophiuchus</i>	5.34	G	- 70.7	+17.0	- 53.8

These stars have small proper motions.

Messrs. Woodgate and Baines took part in exposing the plates. The measures were made by the writer.

ROYAL OBSERVATORY, CAPE OF GOOD HOPE
December 31, 1917

THE GENERAL MAGNETIC FIELD OF THE SUN
APPARENT VARIATION OF FIELD-STRENGTH WITH LEVEL IN
THE SOLAR ATMOSPHERE¹

By G. E. HALE, F. H. SEARES, A. VAN MAANEN, AND F. ELLERMAN

The preliminary results of a study of the Zeeman effect due to the general magnetic field of the sun have been given in a previous paper.² With the aid of suitable polarizing apparatus, used in conjunction with the 75-foot spectrograph of the 150-foot tower telescope, four lines in the third-order spectrum of an excellent Michelson grating were found to show displacements corresponding in sign and agreeing closely in magnitude with theoretical values calculated for a uniformly magnetized sphere. The extreme minuteness of the displacements, usually less than a thousandth of an angstrom, led us to defer final acceptance of the provisional conclusions until they could be rigorously tested by additional measures. The present paper contains results which amply confirm those previously published, and reveal a relation between the intensity of the sun's general field and the character of the spectral lines used for its determination. The stronger lines give smaller values of the field-strength, and, since the intensity of a line depends upon the level in the solar atmosphere at which it originates, it is natural to interpret these differences in field-strength as a consequence of differences in level.

In the original paper only four lines were shown to have displacements attributable to the general field of the sun. A number of others, mainly stronger lines known from laboratory investigations to have large Zeeman separations, showed no corresponding solar displacements. Fortunately an explanation of this apparent contradiction was offered in the circumstance that the displaced lines probably originate at a low level in the solar atmosphere, while

¹ *Contributions from the Mount Wilson Solar Observatory*, No. 148.

² Hale, *Mt. Wilson Contr.*, No. 71; *Astrophysical Journal*, 38, 27, 1913. See also Hale, *Terrestrial Magnetism*, 17, 173, 1912.

the others correspond to higher levels where the field is too weak to be detected.

Although the hypothesis that the general field decreases rapidly in intensity with increasing elevation has usually proved a reliable guide in the choice of additional lines, caution must be exercised in accepting this interpretation of the differences in field-strength, inasmuch as the measures may perhaps be affected by systematic errors depending upon line-intensity. The question is one of much complexity, and the relevant evidence now available is presented in a later section. Although a final answer cannot now be given, this uncertainty in no wise affects the two main results of the investigation. So far as there may be doubt, it is associated with the interpretation of the second result—the relation between field-strength and line-intensity—and, perhaps involved with this, is the puzzling circumstance that certain lines having large displacements in the third-order spectrum show little or no displacement in the second order.

Before proceeding to a discussion of the observations, certain details concerning the measurement of the displacements require consideration.

I. CONFIRMATION OF DISPLACEMENTS BY OTHER OBSERVERS

The observations and measures have always been arranged so as to avoid vitiating influences that might arise from a knowledge of the observing conditions. Nevertheless we have considered it of the utmost importance to obtain all possible confirmation of the measured displacements, which are so small that it is difficult to obtain definite evidence of their reality. The lines are wide in comparison with their shifts, and when measures are first undertaken the accidental errors are usually so large as to mask completely the quantities to be observed, which even after much practice remain for many observers below the limit of perception. Thus five members of the Observatory's staff have made more or less extensive series of measures without obtaining a positive result. On the other hand an equal number of other members of our staff have produced evidence of the objectivity of the displacements and of their agreement with the

hypothesis that the sun behaves approximately as a magnetized sphere.

Besides those whose measures were described in *Mount Wilson Contribution*, No. 71, Miss Richmond and Miss Felker of the Com-

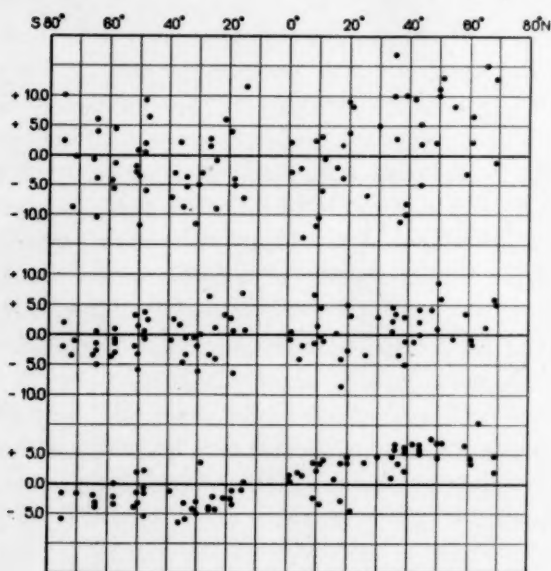


FIG. 1.—Measures by Miss Richmond (upper series), Miss Felker (middle series), and van Maanen on $\lambda 5856.312$. Abscissae are heliographic latitudes; ordinates are values of the displacements, the unit being 0.001 mm. The number of settings of the micrometer in the middle series is twice that for the upper series.

TABLE I
SUMMARY OF MEASURES ON $\lambda 5856.312$

LIMITS OF LATITUDE	MEAN LATITUDE	MEAN Δ		
		Richmond	Felker	van Maanen
60° N–30° N....	47° N	+3.6 \pm 1.1 (24)	+1.8 \pm 0.5 (24)	+5.2 \pm 0.3 (23)
26 N–29 S....	1 S	–1.2 \pm 0.8 (28)	0.0 \pm 0.5 (28)	0.0 \pm 0.4 (28)
30 S–65 S....	49 S	–2.4 \pm 0.7 (29)	–1.2 \pm 0.3 (29)	–2.7 \pm 0.3 (21)

puting Division have made trial series which confirm the general character of the results. These measures, which were on $\lambda 5856.312$ (Fe, 2), are illustrated in Fig. 1, and a summary is given in Table I.

Appended to each mean displacement in the table is its probable error, the unit being 0.001 mm; the number of values included in the mean is added in parentheses. The accidental errors are large (Miss Richmond's series includes half as many settings as that by Miss Felker) and the mean displacements are systematically smaller than those by van Maanen; but for the 45° regions the algebraic signs are all correct, and the means themselves are sufficiently in excess of their probable errors to make the results of significance.

2. MEASUREMENT OF DISPLACEMENTS WITH KOCH'S REGISTERING MICROPHOTOMETER

In the earlier work two forms of measuring machine were employed: a comparator of the ordinary type, with fixed cross-hair, and a parallel-plate micrometer, with which adjacent sections of the displaced line can be brought into alinement by inclining a plane-parallel strip of glass. At the outset the results of different observers with the ordinary micrometer were frequently in disagreement, and it was in the hope of avoiding such discrepancies that the parallel-plate machine was tried. This has proved so satisfactory that all of van Maanen's measures have been made with it. His results for the spectra measured by Miss Lasby with the comparator agree well with hers in sign, although there is a marked systematic difference in magnitude.¹

A promising means of avoiding systematic errors of measurement is offered by Koch's registering microphotometer, which automatically records the distribution of density in the photographic image of a spectral line. The photograph of the line is moved at a uniform rate across a narrow slit, through which light from a constant source falls upon the sensitive electrode of a photo-electric cell. The variations in electromotive force, caused by changes in the intensity of the transmitted light, produce horizontal displacements of the filament of a string electrometer connected with the other electrode. The image of a small section of the filament projected on a photographic plate, moved vertically by the same clock that carries the negative across the slit, traces a record of the intensity-curve of the line (see Fig. 2). The records may be used to determine

¹ *Mt. Wilson Contr.*, No. 71, p. 63; *Astrophysical Journal*, 38, 87, 1913.

the relative positions of spectral lines, as well as the distribution of intensities within them, and thus afford a means of measurement independent of the personal errors that affect observations made with a comparator.

The instrument employed, which was kindly loaned by Professor Röntgen from the collection of the University of Munich for use in Pasadena during Professor Koch's visit in 1913, was not designed for the measurement of extremely small displacements, and it was doubtful whether the necessary degree of precision in the relative motion of the negative and the recording plate could be counted upon.

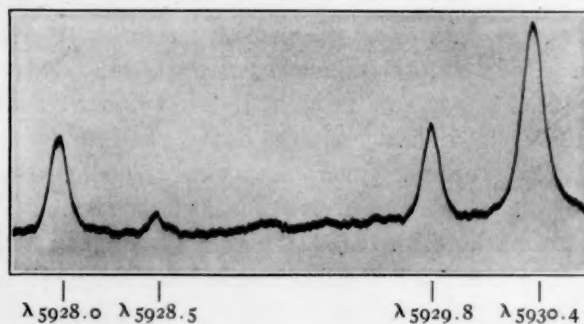


FIG. 2.—Intensity-curve registered by the Koch microphotometer used in measuring the displacements of $\lambda 5928.013$ and $\lambda 5929.898$ with respect to $\lambda 5930.406$, which is not displaced by the sun's general field. The atmospheric line $\lambda 5928.510$ is not suitable as a reference line because of its low intensity. Cut is 0.72 original curve.

Three solar lines were first studied, two of which ($\lambda 5928.013$ and $\lambda 5929.898$) had been found to be subject to displacement, while one ($\lambda 5930.406$) was not appreciably affected and thus furnished a reference mark to which the other lines could be referred.¹ Although the range of spectrum is only 2.4 Å, Professor Koch was compelled to limit the ratio between the movements of the negative and the recording plate to 7.65, instead of using the more advantageous value of 46.4 applicable to very small fields. Curves were measured for two general-field plates representing direct and reverse records of each of four adjoining strips of the

¹ The atmospheric line $\lambda 5928.510$ would be preferable for this purpose, but its intensity-curve cannot be satisfactorily measured.

quarter-wave plate. One of these curves is reproduced in Fig. 2. Pairs of abscissae corresponding to ordinates increasing by one millimeter were measured to hundredths of a millimeter. Two reductions were made. In the first, all the measures were used, averaging fifteen pairs for λ 5928, thirteen for λ 5929, and thirty-one for λ 5930; in the second, only the ten pairs corresponding to ordinates common to all three lines were employed. The means of the abscissae give the positions of the lines, from which the displacements in Table II were derived.

TABLE II
COMPARISON OF MEASURES

LINE	PLATE	KOCH'S REGISTERING PHOTOMETER		VAN MAANEN PARALLEL-PLATE MICROMETER
		First Reduction	Second Reduction	
λ 5928.013.....	T'193a	-2.7 μ	-5.9 μ	-0.7 μ
	193b	+9.3	+8.4	+2.6
λ 5929.898.....	193a	-7.3	-7.9	-10.3
	193b	+6.6	+7.2	+6.2

The differences between the two reductions indicate the uncertainty of the results obtained with the registering photometer. For the shorter interval (λ 5929.898) the measures are in approximate agreement with each other and with those of van Maanen. For the longer interval the inaccuracies of the driving mechanism are naturally reflected in the results.

As a further test, measures were made with the Koch photometer on λ 6302.709 (Fe, 5), which was referred to the neighboring atmospheric line λ 6302.975. Here it was possible to use the larger magnification of 46.4, although owing to the inherent uncertainties it is doubtful if any greater precision was obtained.

Twenty density-curves covering four adjacent spectra were registered from plate T'337a. As before, two reductions were made, one including all the abscissae for each curve, the other only those for ordinates common to the two curves. The results are

First reduction, $\Delta = +1.9 \mu$

Second reduction, $\Delta = +3.0$

From the original negative van Maanen had previously found $+2.8 \mu$. The uncertainty of the results from the Koch machine is rather large in this case, and the agreement with van Maanen's value, though probably of no real significance, is better than might have been expected.

These preliminary results indicate that a specially designed microphotometer should afford measures of great value, particularly for the study of systematic personal errors. As soon as opportunity permits, the instrument recently constructed in our shops will be used for a study of this question.

3. THE OBSERVATIONS

The measures considered in the present discussion depend on the groups of plates designed as Series IV, V, VI, VII, and VIII. The details, as well as the general method of observation, were the same as those previously described. The photographs of the third-order spectrum were made by Ellerman personally or else under his immediate supervision with the 75-foot spectrograph of the 150-foot tower telescope, the 43-cm solar image being used throughout. In view of the high excellence required for successful use, plates have frequently been rejected without measurement; insufficient or excessive contrast and irregularities in density were the most common reasons for rejection.

The record of observations for Series IV was published in *Mount Wilson Contribution*, No. 71, the original investigation having been partly based upon this series. Since Series V–VIII usually include only two days ($\lambda 5247$, Series VII, and $\lambda 4406$ and $\lambda 4418$, Series VIII, were observed on three days, and $\lambda 4421$, Series VIII, on four days), the chronological list of plates for these series is omitted, the essential data being collected in Tables III–VIII.

The observations given in this paper were not intended for an exhaustive study of the sun's magnetic field, but rather as a means of checking the earlier results and of selecting lines suitable for investigating the position of the magnetic axis and other related questions. The measures have therefore been restricted to the regions of maximum displacement near 45° N. and S. latitude. Most of the lines combine the characteristics of low or moderate

solar level and large laboratory separation, although several showing displacements in the sun have been included whose laboratory separations have not yet been determined.

TABLE III
LINES SHOWING MAGNETIC DISPLACEMENT

SERIES	ROWLAND			PLATES	DATES OF OBSERVATION	D	ALGEBRAIC SIGN OF Δ		ZERO VALUES
	λ	El.	Int.				Right	Wrong	
IV.....	5831.821	Ni	1	T' 255-290	1913 Jan. 29-Feb. 18	-6.4	94	28	3
	5856.312	Fe	2	264-290	Jan. 31-Feb. 18	-6.6	66	8	1
	5928.013	Fe	2	258-290	Jan. 30-Feb. 18	-6.5	75	18	2
	6007.540	Ni	1	318-333	June 10-20	+1.7	54	17	1
V.....	6039.953	V	0	"	"	"	79	6	0
	6079.227	Fe	2	"	"	"	94	12	2
	6111.290	Ni	2	324-333	"	"	53	13	0
	6119.740	V	1	318-333	"	"	74	9	0
	6129.190	Ni	1	"	"	"	67	13	0
	6149.458	Fe*	2	"	"	"	65	10	1
	6173.553	Fe	5	"	"	"	85	5	0
VII.....	5247.737	Cr	2	424-440	Sept. 9-11	+7.2	171	12	1
	5250.817	Fe	3	431-436	Sept. 9-10	"	48	6	0
	5253.633	Fe	2	"	"	"	49	5	0
	5263.486	Fe	4	"	"	"	47	5	2
	5300.929	Cr	2	430-435	"	"	45	4	1
	5304.355	Cr	0	"	"	"	36	8	3
	5328.515	Cr	2	"	"	"	46	4	1
	5329.329	Cr	3	"	"	"	42	8	2
	5329.975	Cr	0	"	"	"	40	8	0
	5340.639	Cr	0	"	"	"	37	8	0
VIII.....	5348.511	Cr	4	"	"	"	46	6	1
	4406.810	V	2	463-464	Nov. 10	+3.3	30	3	1
	4418.499	Ti	1	479-480	Dec. 1, 5	+0.5	36	0	0
	4421.733	V	0	463-480	Nov. 10-Dec. 5	49	19	0
	4430.785	Fe	3	463-480	Nov. 10-Dec. 5	23	2	1
	4438.006	V	0	463-464	Nov. 10	+3.3	14	4	1

* Unidentified by Rowland.

The twenty-six new lines for which appreciable displacements have been detected are listed in Table III, with dates of observation, limiting plate numbers, etc. The table also contains $\lambda 5831.821$; the general behavior of this line was indicated in *Mount Wilson Contribution*, No. 71, but the individual displacements are given in this paper for the first time. The last three columns of Table III show that a large majority of the algebraic signs accord with the

hypothesis that the displacements are caused by a general magnetic field. Indeed, but few of the disagreeing cases can be said to present contradictions, for frequently the real displacements are so small that a measured value may show a wrong sign without deviating excessively from the truth.

TABLE IV
LINES FROM SERIES IV-VIII WHICH SHOW NO DISPLACEMENT

SERIES	ROWLAND			ALGEBRAIC SIGN OF Δ		ZERO VALUES
	λ	El.	Int.	Right	Wrong	
IV.....	5804.681	Fe	0	18	20	0
	5838.592	Fe	1	19	19	0
	5848.342	Fe	3	15	16	1
	5892.920	Fe	00	7	9	0
	5905.895	Fe	4	15	18	6
	5916.475	Fe	3	25	31	3
V.....	5991.600		2	22	25	4
	5998.002	Fe	2	23	26	3
	6005.770	Fe	1	31	38	4
	6012.450	Ni	1	22	16	1
	6042.315	Fe	3	31	26	3
	6081.665	V	0	21	24	1
VI.....	6142.700		1	24	25	8
	6455.820	Ca	2	32	38	2
	6496.688	Fe	2	41	34	3
	6597.807	Cr	1	32	44	0
VII.....	5224.471	Ti	0	22	21	3
VIII.....	4404.433	Ti	1N	10	9	0

All the measures discussed have been made by van Maanen with the parallel-plate micrometer. The results, collected for each line according to latitude, are given in Tables V-VIII. Those for Series VIII are less numerous than those for the other series, but the photographs are generally of excellent quality, and compensate in some degree for the smaller number of displacements measured. The results for λ 4421 from three plates of this series (sixteen values of Δ) were, however, rejected after measurement because of discordances due probably to lack of proper density. The line is weak and generally difficult of measurement. The rejected values do not appear in Table VIII.

TABLE V
DISPLACEMENTS FOR LINES OF SERIES IV

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
$\lambda 5831.821$			$\lambda 5831.821-Cont.$			$\lambda 5831.821-Cont.$		
N. 71°...	276a	+0.7	0°...	290a	-1.0	S. 60°...	255a	-3.0
70°...	274a	+1.3	0°...	290b	0.0	60°...	255b	-4.3
68°...	272a	-1.2	S. 3°...	290a	+0.7	60°...	264a	-1.7
68°...	273b	+2.0	3°...	290b	+2.0	60°...	264b	-2.3
64°...	271b	+4.0	7°...	290a	+4.3	62°...	261a	-0.7
63°...	274a	+3.0	7°...	290b	+3.6	62°...	261b	-1.7
60°...	273b	+4.0	12°...	290a	+2.3	63°...	258a	-3.3
59°...	276a	+1.0	12°...	290b	(+6.9)	63°...	258b	-3.6
58°...	271b	+1.3	17°...	290a	+2.6	65°...	265a	-4.6
58°...	272a	-0.7	17°...	290b	(+4.3)	65°...	265b	-1.7
50°...	273b	+0.5	19°...	283a	-1.0	66°...	255a	-5.8
50°...	274a	+1.5	21°...	290a	+0.8	66°...	255b	-1.8
49°...	276a	(-4.1)	21°...	290b	-1.0	66°...	264a	-5.3
47°...	271b	(+7.8)	22°...	283a	-0.3	68°...	264b	-2.0
47°...	272a	+0.7	28°...	283a	-1.0	72°...	255b	-3.8
45°...	274a	+0.3	31°...	283a	-2.0	72°...	265a	-2.6
44°...	273b	+1.5	35°...	283a	-0.2	72°...	265b	-2.1
44°...	276a	+1.7	37°...	283b	-0.7	73°...	255a	(-6.3)
42°...	271b	+1.2	39°...	261b	-0.3	73°...	264a	+2.3
41°...	272a	+5.0	41°...	283b	-0.3	73°...	264b	-1.7
40°...	273b	+0.5	42°...	258a	-2.0	79°...	255b	-2.6
40°...	274a	+2.6	42°...	258b	0.0	79°...	264a	-4.1
38°...	276a	+1.8	42°...	261a	(+5.3)	79°...	264b	+0.3
37°...	271b	-1.0	44°...	261a	(+6.3)	82°...	265a	(-6.9)
36°...	272a	-0.7	44°...	261b	-3.0	S. 82°...	265b	-2.3
36°...	273b	+3.3	46°...	258a	-2.6			
36°...	274a	+2.8	46°...	258b	-4.1			
35°...	271b	-0.3	47°...	255b	+0.8	$\lambda 5856.312$		
30°...	275b	+2.0	47°...	264b	-3.1	N. 69°...	273b	+4.8
27°...	275a	-2.0	47°...	265b	+1.0	69°...	274a	+2.0
26°...	275b	+2.0	48°...	265a	-4.0	66°...	272a	(+10.2)
21°...	288a	+0.8	48°...	283b	-0.7	61°...	273b	+3.6
21°...	288b	+1.7	49°...	255a	-2.0	61°...	274a	+4.0
20°...	275a	+2.3	51°...	264a	(-7.3)	59°...	272a	+6.4
20°...	275b	+1.0	51°...	264b	-1.8	51°...	274a	+6.9
17°...	288a	-3.0	51°...	265a	-3.6	50°...	273b	+6.9
17°...	288b	0.0	51°...	265b	-1.0	50°...	276a	+4.3
16°...	275a	+1.2	51°...	283b	+2.6	48°...	272a	+7.4
16°...	275b	+1.3	52°...	255a	-3.0	44°...	273b	+6.3
14°...	288a	+0.7	52°...	255b	-5.6	44°...	274a	+5.8
14°...	288b	+3.6	52°...	258a	-5.6	44°...	276a	+5.3
12°...	275b	+4.0	52°...	258b	-5.9	42°...	272a	+6.6
10°...	275a	+1.3	52°...	261a	-1.5	39°...	273b	+5.9
10°...	288a	-0.3	52°...	261b	-3.1	39°...	274a	+2.1
10°...	288b	+1.5	56°...	258a	+2.6	39°...	276a	+5.6
9°...	275b	(+6.3)	56°...	258b	-1.2	37°...	272a	+3.3
5°...	288a	+1.3	57°...	261a	-0.3	36°...	273b	+6.3
5°...	288b	+2.1	57°...	261b	+0.5	36°...	276a	+5.9
1°...	288a	+1.3	59°...	265a	(+5.6)	35°...	272a	+1.0
N. 1°...	288b	+1.7	S. 59°...	265b	-4.6	35°...	274a	+4.5
						N. 30°...	275b	+4.5

TABLE V—Continued

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 5856.312—Cont.			λ 5928.013			λ 5928.013—Cont.		
N. 26° ...	275b	+3.6	N. 69° ...	276a	+3.6	S. 30° ...	290a	-1.3
21 ...	288a	(-4.8)	68 ...	273b	+4.3	30 ...	290b	-2.8
20 ...	275b	+3.5	67 ...	271b	+5.3	31 ...	283a	-2.3
20 ...	288b	+4.3	66 ...	272a	+6.8	34 ...	290a	-6.3
18 ...	288a	(-3.0)	62 ...	273b	+4.0	34 ...	290b	-5.4
18 ...	288b	+3.6	61 ...	276a	+4.0	35 ...	283a	-4.8
16 ...	275b	+0.7	59 ...	272a	+5.6	35 ...	283b	-1.0
12 ...	275b	+4.0	57 ...	271b	+3.6	39 ...	283b	+2.3
11 ...	288a	-3.3	50 ...	273b	+3.6	42 ...	258a	-6.6
11 ...	288b	+3.3	50 ...	276a	+5.9	42 ...	258b	+1.5
10 ...	275b	+2.0	48 ...	272a	+3.6	42 ...	261a	-3.6
9 ...	288a	-2.3	44 ...	273b	+4.3	42 ...	261b	-2.3
9 ...	288b	+3.5	44 ...	276a	+4.0	44 ...	261a	-0.3
5 ...	288b	+1.5	42 ...	271b	+1.5	44 ...	261b	-1.7
4 ...	288a	+1.8	42 ...	272a	+4.6	46 ...	258a	+0.3
1 ...	288a	+1.5	39 ...	273b	+1.8	46 ...	258b	+2.5
N. 1 ...	288b	+0.3	39 ...	276a	+2.1	48 ...	264a	-2.6
S. 14 ...	290b	+0.2	37 ...	271b	+2.6	48 ...	264b	+0.7
15 ...	290a	-1.0	37 ...	272a	+1.0	48 ...	283b	(+4.0)
18 ...	290a	-1.2	36 ...	273b	+3.6	50 ...	264a	-1.7
18 ...	290b	-3.3	36 ...	276a	+3.5	50 ...	264b	-2.5
19 ...	283a	-2.6	35 ...	271b	+2.3	50 ...	283b	(+3.3)
21 ...	283a	-2.3	35 ...	272a	+1.3	52 ...	258a	-4.0
24 ...	290a	-4.3	30 ...	275b	+4.3	52 ...	258b	+1.7
24 ...	290b	-2.3	26 ...	275b	+1.3	52 ...	261a	-2.3
26 ...	290a	-4.1	21 ...	288a	+1.8	52 ...	261b	0.0
26 ...	290b	-4.3	21 ...	288b	(-5.6)	56 ...	283b	+2.8
29 ...	283a	(+3.6)	20 ...	275b	+0.8	57 ...	258a	(+5.1)
30 ...	290a	-5.0	17 ...	288a	+5.4	57 ...	258b	-1.5
30 ...	290b	-3.0	17 ...	288b	(-6.6)	57 ...	261a	-5.9
31 ...	283a	-4.3	16 ...	275b	0.0	57 ...	261b	(+3.3)
34 ...	290a	-3.3	12 ...	275b	+2.3	58 ...	264a	-5.4
34 ...	290b	-5.9	12 ...	288a	+4.3	58 ...	264b	-1.3
36 ...	283b	-6.4	12 ...	288b	(-5.0)	63 ...	258b	-2.3
39 ...	283b	-1.3	10 ...	275b	+1.7	63 ...	261a	-0.5
48 ...	264a	-1.3	10 ...	288a	-3.6	63 ...	261b	-4.3
48 ...	264b	-0.7	10 ...	288b	(-8.4)	64 ...	258a	-0.3
48 ...	265a	-5.6	5 ...	288a	+6.3	64 ...	264a	-4.3
48 ...	283b	(+2.1)	5 ...	288b	-4.3	64 ...	264b	-2.0
50 ...	264a	(+1.8)	1 ...	288a	+3.0	68 ...	261b	-1.0
50 ...	264b	-1.7	N. 1 ...	288b	-5.0	70 ...	264a	-0.7
50 ...	265a	-3.3	S. 15 ...	290a	-2.0	70 ...	264b	-3.0
51 ...	283b	-4.0	15 ...	290b	-1.7	S. 76 ...	264a	-4.3
58 ...	264a	0.0	18 ...	283a	-1.8			
58 ...	264b	-2.3	18 ...	290a	-4.6			
58 ...	265a	-3.5	18 ...	290b	-2.3			
64 ...	264a	-4.0	21 ...	283a	-1.3			
64 ...	264b	-3.3	24 ...	290a	-3.3			
64 ...	265a	-2.0	24 ...	290b	-3.0			
71 ...	265a	-1.8	26 ...	290a	-4.0			
75 ...	264a	-1.7	26 ...	290b	-3.3			
S. 75 ...	264b	-5.9	S. 29 ...	283a	-0.3			

TABLE VI
DISPLACEMENTS FOR LINES OF SERIES V

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 6007.540			λ 6007.540—Cont.			λ 6039.953—Cont.		
N. 64°...	333a	+5.0	S. 41°...	319a	-3.0	N. 30°...	324a	+5.6
64....	333b	+7.6	41....	319b	-6.4	39....	324b	+1.7
62....	332a	-1.7	41....	326a	+4.0	39....	332a	+5.1
62....	332b	+8.6	41....	326b	-1.8	39....	332b	+4.6
61....	318a	+3.3	41....	330a	-5.6	39....	333a	+7.1
61....	318b	+3.6	41....	330b	-3.1	39....	333b	+9.2
58....	318a	+0.3	45....	326a	(+5.6)	37....	333a	+6.4
58....	318b	+4.3	45....	327b	-3.6	37....	333b	+6.4
58....	332a	-1.8	46....	330a	-3.3	36....	318b	+6.4
58....	332b	-3.6	46....	330b	-2.1	35....	324a	(-5.3)
57....	333a	+4.1	51....	327a	-6.8	35....	324b	+6.3
57....	333b	+3.5	52....	319a	-3.6	34....	318a	+6.4
50....	333a	+2.8	52....	326a	+5.0	34....	318b	+5.0
50....	333b	+1.3	52....	326b	-4.6	N. 34....	332a	+4.1
49....	318a	+4.6	52....	330a	-2.8	S. 30....	330a	-3.6
49....	332a	+0.3	52....	330b	-5.3	30....	330b	-3.5
49....	332b	-3.1	54....	327b	-3.3	31....	319a	-6.9
45....	332a	-4.0	57....	319a	0.0	31....	319b	-6.6
45....	332b	-1.0	57....	326a	+3.3	32....	327b	-6.1
45....	333a	+5.0	57....	326b	-3.6	35....	319a	-6.9
45....	333b	+4.8	S. 57....	327a	-2.6	35....	319b	-2.3
44....	318a	+3.3	λ 6039.953			35....	326b	-6.9
44....	318b	+4.0	N. 65....	333b	+7.4	35....	327a	-4.0
39....	318a	+3.0	62....	318a	(+11.9)	35....	327b	-5.9
39....	318b	+5.6	62....	332a	+4.0	35....	330a	+0.5
39....	332a	-2.6	62....	332b	+2.8	35....	330b	-7.3
39....	332b	+4.5	60....	324a	(-8.6)	41....	319a	-5.8
39....	333a	+2.6	60....	324b	+3.0	41....	319b	-9.9
39....	333b	+5.0	59....	318a	+7.3	41....	326a	-4.5
35....	318a	+4.6	59....	318b	+3.3	41....	326b	-4.6
35....	318b	+3.5	59....	333a	+4.8	41....	327a	-5.3
35....	332a	-4.6	59....	333b	+9.2	41....	327b	-7.9
35....	332b	+4.6	58....	332a	+6.8	41....	330a	(+10.4)
35....	333a	+2.3	58....	332b	+6.3	41....	330b	-1.3
N. 35....	333b	+3.6	57....	324a	(-5.0)	45....	326a	-6.3
S. 30....	330a	-5.9	57....	324b	+4.0	45....	326b	-5.6
30....	330b	-3.3	50....	318a	+7.3	45....	327a	-4.3
31....	319a	+2.1	50....	318b	+6.4	45....	327b	-1.3
31....	319b	+4.0	50....	324a	+6.6	45....	330a	(+8.6)
31....	327a	+5.0	50....	324b	+5.3	45....	330b	-5.9
31....	327b	-4.6	49....	332a	+7.9	46....	319a	-7.6
34....	330a	-3.0	49....	332b	+4.3	46....	319b	-2.0
34....	330b	-5.0	49....	333a	+4.0	51....	327a	-6.9
35....	319a	+5.4	49....	333b	+3.8	51....	327b	-6.8
35....	319b	-1.3	45....	324a	+9.2	52....	319a	-7.6
35....	327a	+4.0	45....	324b	+3.6	52....	319b	-6.9
35....	327b	-3.1	45....	332b	+7.9	52....	326a	-7.6
37....	326a	(+5.9)	45....	333a	+10.1	52....	326b	-5.6
37....	326b	-4.6	45....	333b	+7.1	52....	330a	-6.6
40....	327a	-3.6	N. 39....	318b	+3.5	52....	330b	-7.6
S. 40....	327b	-6.9				S. 57....	326a	-5.9

TABLE VI—Continued

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 6039.953—Cont.			λ 6079.227—Cont.			λ 6111.290—Cont.		
S. 57°...	326b	-7.3	S. 30°...	319a	-4.0	N. 45°...	333a	+3.3
57°...	330a	-8.9	30°...	319b	-4.6	45°...	333b	+1.8
58°...	319a	-4.0	31°...	327a	-2.6	39°...	324a	+3.3
58°...	319b	-6.3	31°...	327b	-5.4	39°...	332a	+3.6
58°...	327a	-0.2	34°...	326a	-3.5	39°...	332b	+4.3
S. 58°...	327b	-6.8	34°...	326b	-6.9	39°...	333a	+2.6
λ 6079.227			35°...	319a	-5.3	39°...	333b	+2.3
N. 64°...	333a	+5.6	35°...	327a	(-0.2)	35°...	324a	-0.3
64°...	333b	+5.4	35°...	327b	-4.6	35°...	324b	+0.2
63°...	318a	+4.3	40°...	327a	-3.6	35°...	332a	+0.7
63°...	318b	+5.0	41°...	319a	-6.4	35°...	332b	+1.0
62°...	332a	+4.0	41°...	319b	-5.6	35°...	333a	(-3.1)
62°...	332b	+2.6	41°...	326a	-4.0	N. 35°...	333b	+2.3
61°...	324b	+2.0	41°...	326b	-7.6	S. 30°...	330a	(+3.6)
59°...	332a	+3.5	45°...	319a	-5.3	30°...	330b	-1.8
59°...	332b	+1.7	45°...	326a	-6.4	31°...	327a	-3.6
57°...	318a	+4.3	45°...	327a	-4.6	31°...	327b	-2.6
57°...	324a	+4.6	45°...	327b	-6.3	34°...	326a	-1.8
57°...	324b	+4.3	52°...	319a	(-7.9)	34°...	326b	-5.0
57°...	333a	+6.8	52°...	319b	-3.5	35°...	327a	-0.3
57°...	333b	(+7.6)	52°...	326a	-3.0	35°...	327b	-1.3
50°...	318a	+6.1	52°...	326b	-4.0	35°...	330a	+0.3
50°...	318b	+5.6	52°...	327a	-5.4	35°...	330b	-5.4
50°...	324a	+5.6	52°...	327b	-3.6	40°...	327a	-2.3
50°...	324b	+5.6	57°...	319a	-3.6	40°...	327b	-1.7
50°...	333a	+7.4	57°...	319b	-5.6	40°...	330a	(+4.0)
50°...	333b	+5.9	57°...	327a	-5.0	40°...	330b	+0.3
49°...	332a	+4.3	57°...	327b	-3.0	41°...	326a	-0.3
49°...	332b	+3.5	58°...	326a	-5.8	41°...	326b	-2.8
45°...	318a	+4.0	S. 58°...	326b	-5.9	45°...	326a	-1.3
45°...	318b	+5.6	λ 6111.290			45°...	327a	-1.3
45°...	324a	+5.3	N. 64°...	332a	+0.7	45°...	327b	-4.0
45°...	324b	+4.6	64°...	333a	+3.0	45°...	330a	(+4.0)
45°...	332a	+4.1	64°...	333b	+4.3	45°...	330b	+0.7
45°...	332b	+2.3	62°...	332b	+2.3	51°...	330a	+2.3
45°...	333a	+5.6	59°...	332a	+3.3	51°...	330b	-5.9
45°...	333b	+5.8	59°...	332b	+0.2	52°...	326a	-1.7
40°...	324a	+2.6	57°...	324a	+4.3	52°...	326b	-3.0
40°...	324b	+5.3	57°...	324b	+0.5	52°...	327a	-3.0
39°...	318b	+4.0	57°...	333a	+2.1	52°...	327b	-1.7
39°...	332b	+4.1	57°...	333b	-1.3	57°...	327a	-2.5
39°...	333a	+4.0	50°...	324a	+1.0	57°...	327b	-4.5
39°...	333b	+5.9	50°...	324b	-1.7	57°...	330a	+0.2
36°...	318a	+5.1	50°...	333a	+2.3	57°...	330b	-3.6
36°...	318b	+5.9	50°...	333b	+3.5	58°...	326a	-1.3
35°...	324a	+4.6	49°...	332a	+4.0	S. 58°...	326b	-3.3
35°...	324b	+3.0	49°...	332b	+3.0	λ 6119.740		
35°...	332a	+4.6	45°...	324a	+0.7	N. 64°...	332a	+4.3
35°...	332b	+2.6	45°...	324b	+4.0	64°...	333a	(-4.0)
35°...	333a	+3.3	45°...	332a	-1.0	64°...	333b	+1.0
N. 35°...	333b	+5.0	N. 45°...	332b	+0.2	63°...	318b	+6.4

TABLE VI—Continued

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
$\lambda 6119.740$ —Cont.			$\lambda 6119.740$ —Cont.			$\lambda 6129.190$ —Cont.		
N. 62°...	332b	+4.6	S. 40°...	327b	-6.3	N. 44°...	324a	+3.6
60°...	324b	+5.6	40°...	330a	-5.9	44°...	324b	+3.6
59°...	332a	+6.9	40°...	330b	-3.3	40°...	332a	+3.1
59°...	332b	+3.8	41°...	319a	(+7.6)	39°...	324a	+1.3
57°...	318b	+2.6	41°...	319b	-3.6	39°...	324b	+5.6
57°...	324a	(-8.3)	41°...	326a	-3.3	39°...	333a	+4.6
57°...	324b	+2.6	41°...	326b	-5.0	39°...	333b	+1.7
57°...	333a	+5.3	45°...	319a	(+6.3)	38°...	318a	-0.3
57°...	333b	+4.3	45°...	319b	-3.0	35°...	332a	+2.6
50°...	318b	+4.0	45°...	326a	-5.1	35°...	332b	+5.6
50°...	324a	+6.4	45°...	327b	-3.3	35°...	333a	+2.1
50°...	324b	+3.3	45°...	330a	-3.6	34°...	318a	(-4.8)
50°...	333a	+2.1	45°...	330b	-6.9	34°...	324a	+0.3
50°...	333b	+5.3	51°...	330a	-6.1	N. 34°...	324b	+5.4
49°...	332a	+4.5	51°...	330b	-5.9	S. 31°...	319a	+3.0
49°...	332b	+5.8	52°...	327b	-3.6	31°...	319b	-5.3
45°...	318b	+2.3	52°...	326b	-4.6	31°...	330a	-8.3
45°...	324a	+4.3	52°...	326a	-6.3	31°...	330b	-1.7
45°...	324b	+2.0	52°...	319b	-1.2	32°...	326a	-3.8
45°...	332a	+4.0	52°...	319a	(+4.0)	32°...	326b	-3.6
45°...	332b	+5.8	57°...	319a	(+7.3)	35°...	330a	-5.0
45°...	333a	+4.0	57°...	319b	-6.3	35°...	330b	-4.6
45°...	333b	+6.9	57°...	327a	-5.9	36°...	319a	-4.0
39°...	318a	+8.3	57°...	327b	-6.3	36°...	319b	-1.5
39°...	318b	+2.8	57°...	330a	-5.0	36°...	326a	-4.5
39°...	324b	+5.4	58°...	326a	-4.1	36°...	326b	-5.1
39°...	332a	+6.3	S. 58°...	326b	-3.6	36°...	327b	-2.5
39°...	332b	+5.4				41°...	319a	-4.8
39°...	333a	+7.3				41°...	319b	-5.3
39°...	333b	+4.0	$\lambda 6129.190$			41°...	327a	+3.0
36°...	318a	+6.9	N. 64°...	332a	+3.0	41°...	327b	-3.6
36°...	318b	+6.4	64°...	332b	+5.1	41°...	330a	+2.6
35°...	324a	+5.3	63°...	324a	+6.1	41°...	330b	-4.3
35°...	324b	+5.6	63°...	333a	+8.6	42°...	326a	-3.6
35°...	332a	+3.3	63°...	333b	+5.1	42°...	326b	-1.5
35°...	332b	+2.3	62°...	318a	(-6.3)	46°...	319a	-5.6
35°...	333a	+2.3	58°...	332a	+4.3	46°...	319b	-3.0
N. 35°...	333b	+6.9	58°...	332b	+4.3	46°...	326a	-3.0
S. 30°...	319a	-3.3	57°...	324a	+4.0	46°...	326b	-7.3
30°...	319b	-5.3	57°...	324b	+5.3	46°...	327a	-1.0
30°...	330b	-3.0	57°...	333a	-1.7	46°...	327b	-5.3
31°...	327a	-1.0	57°...	333b	+2.0	52°...	319a	-4.3
31°...	327b	-5.4	56°...	318a	(-4.6)	52°...	319b	(+3.5)
34°...	326a	(+3.0)	50°...	332a	+7.3	52°...	326a	-4.3
34°...	326b	-3.0	50°...	332b	+2.3	52°...	326b	-2.5
35°...	319a	(+8.9)	50°...	333a	-1.7	52°...	330a	-4.8
35°...	319b	-5.0	50°...	333b	+3.8	52°...	330b	-3.5
35°...	327a	-4.6	49°...	318a	(-3.3)	53°...	327a	-7.1
35°...	327b	+1.5	48°...	324b	+6.8	53°...	327b	+1.0
35°...	330a	-3.0	45°...	332a	+2.6	58°...	319a	-5.4
35°...	330b	-3.1	45°...	333a	+3.6	58°...	319b	-4.0
S. 40°...	327a	-5.1	45°...	333b	+2.3	S. 58°...	326a	-2.1
			N. 44°...	318a	-1.8			

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 6120. 190—Cont.			λ 6149. 458—Cont.			λ 6173. 553—Cont.		
S. 58° ...	326b	−5.0	S. 35° ...	330b	−2.8	N. 49° ...	318b	+9.2
58 ...	330a	−4.6	36 ...	319a	−3.3	48 ...	324a	+6.8
58 ...	330b	−6.3	36 ...	319b	−4.6	48 ...	324b	+6.9
59 ...	327a	−5.3	36 ...	326a	−5.8	45 ...	332a	+4.0
S. 59 ...	327b	−5.6	36 ...	326b	−6.3	45 ...	332b	+6.3
λ 6149. 458			36 ...	327b	+0.3	45 ...	333a	+6.8
N. 64 ...	332a	+4.0	41 ...	319a	−4.3	45 ...	333b	+5.8
64 ...	332b	+5.1	41 ...	327a	−4.6	44 ...	318a	+6.6
63 ...	324a	+5.3	41 ...	327b	−5.0	44 ...	318b	+5.1
63 ...	333a	+5.1	41 ...	330a	−4.1	44 ...	324a	(−0.7)
63 ...	333b	+6.8	41 ...	330b	−3.3	44 ...	324b	+5.0
58 ...	332a	+2.6	42 ...	326a	−3.6	40 ...	332a	+5.1
58 ...	332b	+2.0	42 ...	326b	−4.6	39 ...	324a	+4.6
57 ...	324a	+4.1	45 ...	330a	−3.0	39 ...	324b	+6.6
57 ...	324b	+5.9	45 ...	330b	0.0	39 ...	333a	+0.2
57 ...	333a	+1.3	46 ...	319a	−6.6	39 ...	333b	+4.0
57 ...	333b	+3.3	46 ...	319b	−2.6	38 ...	318b	+6.4
56 ...	318a	(−3.8)	46 ...	326a	−3.0	35 ...	332a	+6.1
50 ...	332a	+4.1	52 ...	319b	−1.2	35 ...	332b	+5.0
50 ...	332b	+5.3	52 ...	326a	−4.3	35 ...	333a	+0.7
50 ...	333a	−1.7	52 ...	330a	−3.0	35 ...	333b	+4.8
50 ...	333b	+5.1	52 ...	330b	+2.3	34 ...	318a	+5.3
49 ...	318a	(−4.0)	53 ...	327b	−0.5	34 ...	318b	+4.6
48 ...	324a	+2.6	58 ...	319a	−4.3	34 ...	324a	+4.6
48 ...	324b	+0.3	58 ...	319b	−3.0	N. 34 ...	324b	+7.9
45 ...	332b	+3.5	58 ...	326a	−2.3	S. 31 ...	319a	−4.0
45 ...	333a	−1.0	58 ...	326b	−3.3	31 ...	319b	−4.3
45 ...	333b	+5.3	58 ...	330a	−4.0	31 ...	330a	(+1.6)
44 ...	318a	(−5.0)	58 ...	330b	−7.9	31 ...	330b	−3.3
44 ...	324a	+3.6	59 ...	327a	−4.5	32 ...	326a	−5.9
44 ...	324b	+5.0	S. 59 ...	327b	−4.3	32 ...	326b	−5.4
40 ...	332a	+5.3	λ 6173. 553			35 ...	330a	(+0.6)
40 ...	332b	+7.3	N. 64 ...	332a	+3.0	35 ...	330b	−2.8
39 ...	324a	+3.6	64 ...	332b	+5.3	36 ...	319a	−4.8
39 ...	324b	+3.8	63 ...	333a	+6.6	36 ...	319b	−6.4
39 ...	333a	+3.3	63 ...	333b	+8.2	36 ...	326a	−6.3
39 ...	333b	+2.6	62 ...	318a	+6.3	36 ...	326b	−5.3
38 ...	318a	(−4.6)	62 ...	318b	+7.3	36 ...	327a	−1.2
35 ...	332a	+6.3	58 ...	332a	+5.6	36 ...	327b	−5.6
35 ...	332b	−0.3	58 ...	332b	+4.3	41 ...	319a	−4.8
35 ...	333a	+2.0	57 ...	324a	+1.0	41 ...	319b	−4.3
35 ...	333b	+5.0	57 ...	324b	+2.1	41 ...	327a	−8.1
34 ...	318a	(−3.3)	57 ...	333a	+4.3	41 ...	327b	−3.8
N. 34 ...	324a	+0.8	57 ...	333b	+3.3	41 ...	330a	−0.1
S. 31 ...	319a	−1.5	56 ...	318a	+0.2	42 ...	330b	−4.3
31 ...	319b	−2.6	56 ...	318b	+5.8	42 ...	326a	−5.9
31 ...	330a	−5.3	50 ...	332a	+6.6	42 ...	326b	−6.1
31 ...	330b	−1.5</						

TABLE VI—Continued

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 6173.553—Cont.			λ 6173.553—Cont.			λ 6173.553—Cont.		
S. 46°...	326b	-7.3	S. 52°...	330a	(+2.3)	S. 58°...	326b	-7.6
46....	327a	-6.9	52....	330b	-4.6	58....	330a	-2.2
46....	327b	-3.8	53....	327a	-5.3	58....	330b	-6.4
52....	319a	-1.7	53....	327b	-4.3	59....	327a	-3.6
52....	319b	-3.6	58....	319a	-6.3	S. 59....	327b	-6.9
52....	326a	-5.0	58....	319b	-5.0			
S. 52....	326b	-6.1	S. 58....	326a	-7.1			

Since the compound quarter-wave plate was sometimes used in the normal (+) and sometimes in the inverted (−) position, and since in nearly all cases the measurer was ignorant of its position and also of the hemisphere under observation, personal bias has been eliminated. The signs of displacements observed with the inverted quarter-wave plate have been changed, so that all results refer to the normal position of the plate.

Besides the lines given in Table III, others have also been measured. A list of these, which show no sensible displacement, is in Table IV. The close equality in the numbers of right and wrong signs is in striking contrast with the preponderance of correct signs in Table III.

4. DETERMINATION OF MAXIMUM DISPLACEMENT FOR DIFFERENT LINES

The first step in the calculation of the sun's general field is the determination of the displacement at $\phi = 45^\circ$ from the measures in Tables V–VIII. The displacements of a normal Zeeman triplet produced by the sun's field, assumed to be that of a uniformly magnetized sphere, may be represented by¹

$$k\Delta = A \cos i + B \sin i \cos \lambda \quad (1)$$

in which

Δ = displacement of spectral line,

k = a constant depending upon the strength of field, the magnetic separation of the line, and the units employed,

i = inclination of sun's magnetic axis to axis of rotation, and

λ = heliographic longitude of magnetic axis.

¹ Seares, *Mt. Wilson Contr.*, No. 72; *Astrophysical Journal*, 38, 99, 1913.

TABLE VII

DISPLACEMENTS FOR LINES OF SERIES VII— $\lambda 5247.737$

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
N. 70°...	424a	+4.0	N. 50°...	429a	+5.9	S. 25°...	440b	-5.3
70°...	424b	+6.3	50°...	429b	+3.6	29°...	433a	-5.8
70°...	426a	+5.1	50°...	430a	+5.6	29°...	433b	(+4.3)
70°...	426b	+4.3	50°...	430b	+6.3	29°...	434a	-4.3
70°...	427a	+4.1	50°...	431a	+5.9	29°...	434b	-2.8
70°...	427b	+5.3	50°...	431b	+3.6	29°...	435a	-2.6
70°...	428a	(-4.3)	50°...	432a	+4.1	29°...	435b	-5.6
70°...	428b	+5.1	50°...	432b	+2.6	29°...	436a	-6.1
60°...	429a	+5.6	46°...	424a	+3.3	29°...	436b	-4.0
60°...	430a	+3.6	46°...	424b	+5.0	29°...	437a	-5.3
60°...	430b	+2.0	46°...	426a	+3.3	29°...	437b	-5.6
60°...	431a	+4.3	46°...	427a	+0.3	29°...	438a	-5.0
60°...	431b	+4.8	46°...	427b	+2.6	29°...	438b	-7.3
60°...	432a	(+8.3)	46°...	428a	(-5.6)	29°...	439a	-4.3
60°...	432b	+5.9	46°...	428b	+2.6	29°...	439b	-4.0
63°...	424a	+2.6	45°...	429a	+5.3	30°...	440a	-7.1
63°...	424b	+5.1	45°...	429b	+3.6	35°...	433a	-6.6
63°...	426a	+5.3	45°...	430a	+5.9	35°...	433b	(+6.3)
63°...	426b	+3.6	45°...	430b	+5.0	35°...	434a	-5.0
63°...	427a	+2.5	45°...	431a	+4.3	35°...	434b	-6.6
63°...	427b	+7.6	45°...	431b	+3.6	35°...	435a	-5.6
63°...	428a	(-3.6)	45°...	432a	+4.8	35°...	435b	-4.3
63°...	428b	+3.6	45°...	432b	+6.3	35°...	436a	-4.0
62°...	429a	+4.3	41°...	424a	+4.0	35°...	436b	-5.3
62°...	430a	+3.3	41°...	424b	+3.0	35°...	437a	-4.0
62°...	430b	+6.1	41°...	426a	+1.7	35°...	437b	-5.1
62°...	431a	+6.6	41°...	426b	+6.3	35°...	438a	-4.6
62°...	431b	+5.4	41°...	427a	+4.6	35°...	438b	-4.5
62°...	432a	+4.5	41°...	427b	+4.6	35°...	439a	-3.8
62°...	432b	+6.9	41°...	428b	+4.3	35°...	439b	-3.3
56°...	424a	+4.3	40°...	429a	+2.1	35°...	440a	-5.0
56°...	424b	+2.6	40°...	429b	+4.0	35°...	440b	-6.1
56°...	426a	+3.3	40°...	430a	+3.5	39°...	433a	-6.3
56°...	426b	+2.6	40°...	430b	+4.8	39°...	433b	(+4.1)
56°...	427a	+4.6	40°...	431a	+5.6	39°...	434a	-4.6
56°...	427b	+4.6	40°...	431b	+4.0	39°...	434b	-6.3
56°...	428a	(-5.4)	40°...	432a	+5.3	39°...	435a	-3.6
55°...	429a	+5.4	N. 40°...	432b	+3.8	39°...	435b	-5.6
55°...	429b	+5.3	S. 25°...	433a	-3.0	39°...	436a	-5.6
55°...	430a	+4.1	25°...	433b	(+5.3)	39°...	436b	-5.6
55°...	430b	+2.6	25°...	434a	-5.1	39°...	437a	-1.0
55°...	431a	+3.3	25°...	434b	-1.8	39°...	437b	-3.3
55°...	431b	+6.4	25°...	435a	-5.4	39°...	438a	-4.3
55°...	432a	+5.6	25°...	435b	-3.3	39°...	438b	-5.6
55°...	432b	+5.4	25°...	436a	-2.6	39°...	439a	-4.0
51°...	424a	+2.3	25°...	436b	-6.3	39°...	439b	-7.6
51°...	424b	+4.5	25°...	437a	-1.7	40°...	440a	-4.6
51°...	426a	+3.3	25°...	437b	-5.3	40°...	440b	-3.5
51°...	426b	+3.5	25°...	438a	-4.8	47°...	433a	-2.3
51°...	427a	+5.4	25°...	438b	-4.1	47°...	433b	(+2.8)
51°...	427b	+5.9	25°...	439a	-2.0	47°...	434a	-4.8
51°...	428a	(-3.0)	25°...	439b	-4.8	47°...	434b	-6.4
N. 51°...	428b	+4.6	S. 25°...	440a	-4.0	S. 47°...	435a	-3.6

TABLE VII—Continued

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
S. 47°...	435b	-3.0	S. 48°...	440a	-6.3	S. 51°...	437b	-6.6
47....	436a	-3.3	48....	440b	-4.6	51....	438a	-5.6
47....	436b	-5.0	51....	433a	-6.3	51....	438b	-3.6
47....	437a	-2.3	51....	433b	(+4.0)	51....	439a	-5.6
47....	437b	-6.6	51....	434a	-4.6	51....	439b	-5.0
47....	438a	-4.6	51....	434b	-7.3	52....	440a	-3.1
47....	438b	-5.4	51....	435a	-6.6	S. 52....	440b	0.0
47....	439a	-4.3	51....	435b	-7.1			
S. 47....	439b	-5.6	S. 51....	437a	+0.3			

Lat.	Plate	Δ			Lat.	Plate	Δ		
		$\lambda 5250$	$\lambda 5253$	$\lambda 5263$			$\lambda 5250$	$\lambda 5253$	$\lambda 5263$
N. 69°..	431a	+2.0	+5.1	+1.7	S. 25°..	435b	-1.0	+0.3	-2.3
69....	431b	+4.5	(-3.1)	+2.6	25....	436a	-0.7	-0.3	-2.3
69....	432a	+1.8	+2.3	+3.8	29....	434a	-3.0	-3.0	+0.3
69....	432b	-1.3	+1.3	+0.3	29....	434b	(+3.6)	+1.3	-0.2
62....	431a	+3.6	+2.0	(+5.0)	29....	435a	+0.3	-0.3	-1.5
62....	431b	+4.0	+4.3	+1.3	29....	435b	-3.6	-3.6	-2.3
62....	432a	+1.3	+1.7	(-1.3)	29....	436a	+0.7	-3.5	-2.8
62....	432b	+1.8	+4.1	+1.0	35....	434a	-3.6	-3.8	-2.1
55....	431a	+0.7	+0.7	0.0	35....	434b	-2.3	-1.7	-3.3
55....	431b	+2.0	+3.5	+1.7	35....	435a	-3.6	-3.1	-1.3
55....	432a	+3.3	+2.6	+3.1	35....	435b	-2.6	-0.7	(+1.2)
55....	432b	+1.3	+4.3	+2.5	35....	436a	-2.6	-1.3	+0.7
50....	431a	+1.7	+0.3	+2.3	39....	434a	-3.0	-1.2	-3.6
50....	431b	+4.3	+2.6	0.0	39....	434b	-2.0	-3.5	-1.0
50....	432a	+4.3	+3.6	+2.3	39....	435a	-2.6	(+1.3)	-2.0
50....	432b	+1.0	+3.1	+3.3	39....	435b	-2.1	-5.3	-1.7
45....	431a	+2.3	+2.0	+1.3	39....	436a	-3.6	-3.3	-4.0
45....	431b	+2.8	+3.3	+2.3	47....	434a	-2.3	-4.0	-1.3
45....	432a	+3.3	+2.3	+1.2	47....	434b	-4.0	-0.7	-0.3
45....	432b	+5.3	+2.6	+2.3	47....	435a	+0.2	-3.6	-4.1
40....	431a	+3.6	+2.1	+1.0	47....	435b	-0.3	-3.6	-5.0
40....	431b	+3.0	+1.0	+2.0	47....	436a	-4.3	-2.3	-3.0
40....	432a	+1.8	+2.6	+2.1	51....	434a	-0.8	-2.6	-1.0
N. 40....	432b	+0.2	+1.8	+3.6	51....	434b	-5.6	(-6.3)	(-6.3)
S. 25....	434a	(+2.6)	+0.3	+1.0	51....	435a	-1.7	-2.3	-3.6
25....	434b	-2.6	-1.0	-1.2	51....	435b	-3.0	-2.6	-1.0
S. 25....	435a	-2.3	-4.0	-1.8	S. 51....	436a	-3.6	-1.0	-3.3

TABLE VII—Continued

Lat.	Plate	Δ						
		$\lambda 5300$	$\lambda 5304$	$\lambda 5328$	$\lambda 5329.3$	$\lambda 5329.9$	$\lambda 5340$	$\lambda 5348$
N. 69°	430a	+8.3	+2.3	+1.3	+3.6	+4.8	+1.3
69	430b	+3.3
69	431a	+2.0	-0.2	+3.6	-1.0	+0.8	+4.6	-0.5
69	431b	+3.0	0.0	(+5.3)	+2.0	+5.8	+4.3	(+4.6)
69	432a	(+7.6)	(+14.2)	(+5.3)	+2.3	+5.9	+3.0	+1.5
69	432b	+4.6	+4.1	+1.8	+3.1	-2.3	+7.8	0.0
62	430a	+5.0	0.0	+2.6	-0.7	+2.1	+2.3	+0.7
62	430b	+0.7
62	431a	+4.8	+4.1	+3.0	+3.0	+5.0	+4.3	+3.8
62	431b	+5.3	+1.3	+2.8	(-2.3)	+1.3	+2.3	+1.7
62	432a	+2.6	+5.3	+1.3	+2.6	+3.6	+0.6	+2.0
62	432b	+5.0	+7.4	+2.6	-0.7	+6.9	+1.0	+3.6
55	430a	+1.7	(-4.6)	-0.7	+0.7	+2.6	-2.3	+0.7
55	430b	+0.8
55	431a	+3.6	+7.3	-0.5	-0.2	+2.6	-1.0	+1.7
55	431b	+3.8	+5.3	+2.0	+2.3	+1.2	-1.0	+1.5
55	432a	+4.3	+3.5	+4.0	+1.5	+4.0	+2.1	+3.0
55	432b	+1.3	+6.9	+3.6	+2.6	-2.6	+7.6	+1.3
50	430a	+3.3	+2.0	+2.6	+3.6	+5.3	+8.7	+2.8
50	430b	+3.6
50	431a	-0.3	-1.7	+1.8	+2.6	+4.3	+5.3	+0.8
50	431b	+2.8	+8.7	+2.3	+1.7	+0.8	+8.3	+2.3
50	432a	+2.1	+9.2	+0.7	+1.3	-0.3	+2.6	-0.3
50	432b	+5.0	+1.3	+1.2	+2.6	+8.6	+4.0	+2.3
45	430a	0.0	+6.9	+2.0	+1.7	+6.1	+4.6	+4.1
45	431a	+2.0	+4.6	+3.3	+2.6	-2.0	+2.5	+2.6
45	431b	+0.8	+2.0	+1.0	+2.0	+0.5	+8.4	+0.7
45	432a	+3.0	+5.6	+2.1	+1.3	+3.3	(-4.3)	+1.2
45	432b	+2.8	+7.8	+2.3	+2.3	+4.1	+7.9	+0.7
40	430a	+4.0	+5.9	+2.5	+5.3	+1.8	(+4.5)
40	431a	+3.0	0.0	+1.0	+3.0	+4.0	+5.0	+2.8
40	431b	-0.3	+6.3	+1.8	(-2.3)	+5.6	(-4.6)	+2.3
40	432a	+5.3	+3.0	+1.7	+1.8	-2.3	+8.7	+1.0
N. 40	432b	(-1.3)	+1.8	+2.0	0.0	-1.3	(-2.3)
S. 24	434b	(+10.9)
25	434a	-3.0	+2.5	0.0	-2.0	+0.7	-5.0	-1.7
25	434b	-3.6	0.0	-1.0	+0.7	-1.8
S. 25	435a	-2.0	-3.0	-3.6	-5.3	+1.3	-1.3

TABLE VII—Continued

Lat.	Plate	Δ						
		$\lambda 5300$	$\lambda 5304$	$\lambda 5328$	$\lambda 5329.3$	$\lambda 5329.9$	$\lambda 5340$	$\lambda 5348$
S. 25	435 ^b	-2.0	-3.8	(+1.7)
28	434 ^b	+0.8
29	434 ^a	+4.6	-3.3	-2.6	-4.8	-5.0	-1.3
29	434 ^b	-0.7	-0.7	(+2.0)	-0.3	-5.9	-0.3
29	435 ^a	-3.6	-6.6	-2.0	-2.3	-5.6	-10.2	-1.0
29	435 ^b	-2.1	-2.3	-2.1
35	434 ^a	-4.6	-4.5	(-5.3)	-3.6	-4.0	-4.1	-0.3
35	434 ^b	-4.6	-7.4	-3.0	-0.5	-6.3	-0.3	-1.2
35	435 ^a	-3.6	-5.0	-0.5	-2.1	+0.3	+0.3	+0.2
35	435 ^b	-1.0	-1.6	+0.7
39	434 ^a	-3.6	-11.6	+0.7	-2.3	-3.0	-0.2	-2.5
39	434 ^b	-2.8	-3.6	-3.5	-5.0	-4.3	(-12.2)	-2.0
39	435 ^a	-3.0	+0.3	-0.8	-0.3	-8.3	-2.6	-2.0
39	435 ^b	-5.0	-4.5	-1.8
47	434 ^a	+0.5	-8.6	-3.3	-4.3	-0.7	-2.0
47	434 ^b	-5.0	-4.3	-0.5	-1.0	-6.4	-4.6	-0.2
47	435 ^a	-0.2	-11.1	-2.6	-3.5	-3.1	-7.8	-0.3
51	434 ^a	-2.0	-5.6	(+1.8)	-1.0	-2.1	-1.8	-0.7
51	434 ^b	(-7.1)	-6.3	-2.6	-2.5	-9.2	-5.1	(-5.0)
51	435 ^a	-3.1	-3.0	-1.0	+1.5	-3.8	-3.6	-1.2
S. 51	435 ^b	-3.1

The coefficients A and B are defined by

$$\begin{aligned} A &= 3 \sin (2\phi - D) + \sin D \\ B &= 3 \cos (2\phi - D) + \cos D \end{aligned} \quad (2)$$

in which

ϕ = heliographic latitude of the point observed,

D = heliographic latitude of sun's center.

Were D and i both zero, $k\Delta$ would equal $3 \sin 2\phi$, which has zero values at $\phi = 90^\circ$ N., 0° , and 90° S., a maximum at 45° N. and a minimum at 45° S. Since the observed displacement curves are approximately of this character, it follows that i must be a small angle (D is known to be small). This permits the calculation of the field-strength without further knowledge of i or λ , for, applying equation (1) to equal northern and southern values of ϕ ,

$$k(\Delta_n - \Delta_s) = 6 \sin 2\phi (\cos D \cos i + \sin D \sin i \cos \lambda), \quad (3)$$

whence¹

$$\frac{4}{CH_p} (\Delta_n - \Delta_s) = 6 \sin 2\phi, \quad (4)$$

¹ *Mt. Wilson Contr.*, No. 72, p. 11; *Astrophysical Journal*, 38, 109, 1913.

in which

Δ is to be expressed in angstroms, and

H_p = field-strength in gaussian units at sun's magnetic pole,

C = separation of the n -components of the triplet in angstroms for a field of 1 gauss.

The omission of D , i , and λ has introduced an error of the order of i^2 (about 1 per cent), a precision that is ample, for the individual values of Δ are subject to uncertainties of 25 per cent or more.

For the determination of H_p it is convenient to apply equation (4) to the displacements observed at $\phi = 45^\circ$, thus giving

$$H_p = \frac{4\Delta_{45}}{3C} \quad (5)$$

in which Δ_{45} is the mean of the absolute values for $\phi = 45^\circ$ N. and S.

To determine Δ_{45} , equation (1) may be written

$$k\Delta = A \quad (6)$$

whence

$$\Delta_{45} = \Delta \frac{A_{45}}{A} \quad (7)$$

This neglects quantities of the order of i , but a combination of results for the observations in the northern and southern hemispheres, which with few exceptions are symmetrically distributed, reduces the error to one of the second order. In fact, the application of (6) to equal values of ϕ , N. and S., leads directly to (4), which is of this precision. The use of (7) is facilitated by the tabulation of A with the arguments ϕ and D .

To shorten the calculation, means were found for groups of displacements observed for each line in neighboring latitudes, the limits in general being 20° - 29° , 30° - 39° , 40° - 49° , 50° - 59° , 60° - 69° , N. and S. The occasional values of Δ below 20° and above 69° were disregarded, since their contribution to the weight of Δ_{45} would have been insignificant.

The value of Δ_{45} having been found by a least-squares solution (the two hemispheres were treated separately), k was calculated by (6), which was then used for the detection of discordant

TABLE VIII
DISPLACEMENTS FOR LINES OF SERIES VIII

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 4406.810			λ 4418.499—Cont.			λ 4421.733—Cont.		
N. 60°...	470a	+4.0	S. 33°...	463a	-3.0	S. 42°...	479b	-5.3
60°...	470b	+3.0	33°...	463b	-4.3	43°...	463a	-9.6
52°...	470a	+3.5	33°...	464a	-3.8	43°...	463b	-7.3
52°...	470b	-0.3	33°...	464b	-1.7	43°...	464a	-7.6
52°...	480a	+0.2	38°...	463a	-6.3	47°...	478a	-3.0
48°...	470b	+1.0	38°...	463b	-5.1	47°...	478b	-4.6
47°...	480a	+2.5	38°...	464a	(-6.6)	47°...	479a	-4.0
41°...	470a	+3.5	38°...	464b	-0.3	47°...	479b	-5.0
40°...	480a	+5.6	43°...	463a	-3.8	51°...	463a	-5.9
36°...	470a	+3.3	43°...	463b	-3.3	51°...	463b	-3.6
36°...	470b	+3.5	43°...	464a	-1.3	51°...	464a	(-11.4)
35°...	480a	+2.8	43°...	464b	-2.3	53°...	478a	-5.6
31°...	480a	(+7.4)	51°...	463a	-1.3	53°...	478b	-2.8
N. 27°...	480a	+2.3	51°...	463b	-3.3	53°...	479a	-1.2
S. 33°...	463a	-4.8	51°...	464a	-4.1	53°...	479b	-6.9
33°...	463b	-4.5	51°...	464b	-0.2	57°...	463a	+0.5
33°...	464a	-1.3	57°...	463a	-2.3	57°...	463b	-8.9
33°...	464b	(+1.5)	57°...	463b	-4.5	57°...	464a	-2.0
38°...	463a	-4.8	57°...	464a	-1.3	58°...	478a	-8.9
38°...	463b	-4.0	S. 57°...	464b	-2.6	58°...	478b	-2.8
38°...	464a	-3.0	λ 4421.733			58°...	479a	-5.8
38°...	464b	-1.8	N. 60°...	470a	+2.0	S. 58°...	479b	-2.6
43°...	463a	-3.1	60°...	470b	+1.3	λ 4430.785		
43°...	463b	-2.5	52°...	470a	+1.7	N. 52°...	480a	(-0.8)
43°...	464a	-5.0	52°...	470b	+3.0	47°...	480a	+2.3
43°...	464b	-5.9	52°...	480b	+5.0	40°...	480a	+3.3
51°...	463a	-2.6	48°...	470a	+3.3	35°...	480a	+2.0
51°...	463b	+0.2	48°...	470b	+2.3	31°...	480a	+3.0
51°...	464a	-2.3	46°...	480b	+5.6	N. 27°...	480a	+3.6
51°...	464b	-2.0	41°...	470b	+0.3	S. 33°...	463a	-2.5
57°...	463a	-1.7	40°...	480b	+4.3	33°...	463b	-3.0
57°...	463b	0.0	36°...	470a	+4.0	33°...	464a	-1.0
57°...	464a	-2.0	36°...	470b	+1.5	33°...	464b	-2.8
S. 57°...	464b	-5.1	35°...	480b	+5.3	38°...	463a	-5.1
λ 4418.499			31°...	480b	+1.3	38°...	463b	-5.3
N. 60°...	470a	+2.6	N. 27°...	480b	+6.3	38°...	464a	-4.0
60°...	470b	+4.6	S. 32°...	478a	-8.9	38°...	464b	-2.6
52°...	470a	+3.3	32°...	478b	+1.3	43°...	463a	-5.1
52°...	470b	+1.0	32°...	479a	-5.3	43°...	463b	-3.3
52°...	480a	+1.3	32°...	479b	-8.9	43°...	464a	-3.6
48°...	470a	+3.3	33°...	463a	-6.8	43°...	464b	-1.8
48°...	470b	+2.3	33°...	463b	-7.9	51°...	463a	-0.7
47°...	480a	+2.5	33°...	464a	-2.1	51°...	463b	(+2.3)
41°...	470a	+4.8	36°...	478a	-7.6	51°...	464a	-3.1
41°...	470b	+1.2	36°...	478b	(+5.3)	51°...	464b	-2.5
40°...	480a	+5.8	37°...	479a	-5.3	57°...	463a	-1.5
36°...	470a	+2.6	37°...	479b	-5.9	57°...	463b	-3.6
36°...	470b	+3.1	38°...	463a	-6.3	57°...	464a	0.0
35°...	480a	+3.3	38°...	463b	-9.9	S. 57°...	464b	-4.0
31°...	480a	+2.0	38°...	464a	-7.9			
N. 27°...	480a	+4.3	S. 42°...	478a	(+5.0)			

TABLE VIII—Continued

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 4438.006			λ 4438.006—Cont.			λ 4438.006—Cont.		
S. 33°...	463a	+0.3	S. 43°...	463a	-0.5	S. 51°...	464b	-3.1
33....	463b	-3.6	43....	463b	-2.3	57....	463a	-3.3
33....	464a	-2.6	43....	464a	-0.7	57....	463b	-2.3
33....	464b	0.0	43....	464b	-2.0	57....	464a	+0.7
38....	463a	-3.5	51....	463a	-3.3	S. 57....	464b	-4.0
38....	463b	-3.3	51....	463b	+0.3			
S. 38....	464b	(+3.8)	S. 51....	464a	(-9.2)			

observations. Values giving residuals greater than three times the probable error were rejected, and the calculation was then revised for the determination of final values of Δ_{45} and k . Rejected observations appear in parentheses in Tables V–VIII.

The results are given in Table IX, which contains the values of Δ_{45} for each hemisphere without distinction as to sign, their differences and their means, the value of k , the original number of observations within the chosen limits for ϕ , the number rejected, and the probable error of a single observed value of Δ derived from a comparison with the theoretical displacement curve written in the form (6). The unit for Δ_{45} and the probable error is 0.001 mm. The table also indicates, in the last column, the value of one angstrom in millimeters, which is necessary for the transformation of Δ_{45} into angstroms.

In order to utilize all the available data, the results by van Maanen on λ 5929.898, Series I and III, and on λ 5812.139 and λ 5828.097, Series IV,¹ are also given in Table IX. To secure homogeneity these have been rediscussed by the foregoing method, which accounts for small differences between the values in Table IX and those previously published. The only results not included are the fragmentary measures of λ 5812 and λ 5828 in Series I and the results of Series II, which were from second-order spectra.

The consistency of the data is illustrated by the differences in the fifth column of Table IX, which by equation (1) are of the form

$$k(\Delta_{+45} + \Delta_{-45}) = 2 \sin D \cos i + 2 \cos D \sin i \cos \lambda \quad (8)$$

¹ *Mt. Wilson Contr.*, No. 71, p. 63; *Astrophysical Journal*, 38, 87, 1913.

and have the limiting values $2 \sin(D+i)$ and $2 \sin(D-i)$. The numerical results are such that i , as already assumed, cannot exceed a few degrees. Considering the difficulty of measurement

TABLE IX
VALUES OF Δ_{45}
(Unit for Δ_{45} and P.E. is 0.001 mm)

SERIES	A	Δ_{45}				k	No. OBSER.		P.E. $\pm \Delta$	FACTOR
		N.	S.	Diff.	Mean		All	Rej.		
I....	5929.898	3.69	5.02	-1.33	4.36	0.70	71	7	± 2.96	4.91
III...	5929.898	4.77	5.50	-0.73	5.14	0.59	125	9	3.38	4.91
	5812.139	4.96	4.16	+0.80	4.56	0.66	96	10	1.61	4.87
IV...	5828.097	4.44	3.74	+0.70	4.09	0.74	98	11	1.44	4.88
	5831.821	1.48	2.14	-0.66	1.81	1.69	86	6	1.66	4.88
	5856.312	5.36	3.55	+1.81	4.46	0.70	54	5	1.72	4.89
	5928.013	3.70	2.10	+1.60	2.90	1.12	74	5	2.15	4.91
V....	6007.540	2.44	2.31	+0.13	2.38	1.26	72	2	2.51	4.93
	6039.953	6.15	5.79	+0.36	5.97	0.50	85	6	2.17	4.94
	6079.227	4.81	5.07	-0.26	4.94	0.60	74	3	0.99	4.95
	6111.290	1.99	2.18	-0.19	2.08	1.44	66	4	1.49	4.96
	6119.740	4.90	4.42	+0.48	4.66	0.64	83	8	2.10	4.96
	6129.190	3.41	3.83	-0.42	3.62	0.83	80	5	2.08	4.96
	6149.458	3.64	3.73	-0.09	3.68	0.81	76	5	1.79	4.96
	6173.553	5.14	5.10	+0.04	5.12	0.58	90	5	1.58	4.97
VII..	5247.737	4.61	4.89	-0.28	4.75	0.62	184	12	1.50	4.72
	5250.817	2.60	2.50	+0.10	2.55	1.16	54	2	1.14	4.72
	5253.633	2.69	2.39	+0.30	2.54	1.18	54	3	1.14	4.72
	5263.486	2.00	2.06	-0.06	2.03	1.47	54	4	1.03	4.72
	5300.929	2.96	2.96	0.00	2.96	1.01	50	3	1.31	4.74
	5304.355	4.46	6.15	-1.69	5.30	0.58	47	3	2.94	4.74
	5328.515	2.12	2.02	+0.10	2.07	1.44	51	4	1.01	4.75
	5329.329	1.94	2.36	-0.42	2.15	1.40	52	3	1.17	4.75
	5329.975	2.75	3.98	-1.23	3.36	0.92	48	0	2.07	4.75
	5340.639	4.73	3.58	+1.15	4.16	0.73	45	3	2.60	4.75
	5348.511	1.79	1.29	+0.50	1.54	1.98	53	5	0.95	4.75
VIII.	4406.810	2.88	3.13	-0.25	3.00	1.00	34	2	1.28	4.50
	4418.499	3.14	3.04	+0.10	3.09	0.97	36	1	1.10	4.51
	4421.733	3.33	5.74	-2.41	4.54	0.72	52	3	2.12	4.51
	4430.785	3.09	3.04	+0.05	3.06	0.98	26	2	1.01	4.51
	4438.006	2.09	2.09	1.41	19	2	± 1.44	4.51

and the smallness of the unit in which the differences are expressed (1μ), the internal agreement is very good, especially for Series V and VII, which include later measures and plates of somewhat better quality.

5. CONFIRMATION OF THE EXISTENCE OF THE SUN'S GENERAL FIELD

It is appropriate at this point to remark upon the character of the evidence now presented as bearing upon the existence of the sun's general magnetic field. It is scarcely necessary to state that it confirms the results of the preliminary investigation. For example, a moment's inspection of Tables V-VIII shows that the displacements of the twenty-six additional lines agree with those found for the four lines discussed in *Mount Wilson Contribution*, No. 71. The algebraic signs are opposite in the northern and southern hemispheres and give the same magnetic polarity as was deduced from the earlier results. Such differences as occur relate mainly to the amplitude of the displacement-curves, and this is determined by the field-strength at the points in which the different lines originate and the magnitude of their respective Zeeman separations, modified to some extent perhaps by systematic influences depending upon line-intensity.

Comparing the variation of the displacements as a function of latitude with the theoretical behavior of a uniformly magnetized sphere, we find as before an agreement that is within the limits of the uncertainty affecting the measured displacements. Moreover, the small differences in the values of Δ_{45} for the northern and southern hemispheres, which have been collected in Table IX, leave no doubt as to the correctness of the original estimate that the inclination of the magnetic axis to the solar axis of rotation cannot exceed a few degrees.

Added weight is given to these conclusions by the precautions taken to avoid physiological error and by the independent confirmation of the displacements by measurers who were entirely ignorant of the data of observation. But by drawing upon the results of another investigation, which will be published in detail later,¹ we can make the case even stronger.

In the present paper we have been content to indicate that the inclination of the sun's magnetic axis does not exceed a few degrees and may be disregarded without appreciably affecting the results

¹ See, however, a preliminary account in *Mount Wilson Communication*, No. 50; *Proceedings National Academy of Sciences*, 4, 4, 1918.

of the discussion. It is easy, however, to rearrange the fundamental equation (1) in such a way as to derive from suitably arranged observations made on a single day a value of the quantity

$$Y = \tan i \cos \lambda \quad (9)$$

The longitude λ varies from day to day, and a comparison of the values of Y for different days extending over a sufficient interval will enable us to determine the inclination i and the period of revolution P . From an extended series of observations on three chromium lines, λ 5247, λ 5300, and λ 5329, it has been found that the calculated values of Y are actually in close agreement with a periodic function of the form of (9).¹

It is of interest to consider the implications of this result. The curve of displacements defined by equation (1) is very nearly a sine curve. Disregarding D , which is always small, the curve will pass through the origin if i is zero. In general, for $\phi = 0$,

$$k\Delta = -2 \sin D \cos i + 4 \cos D \sin i \cos \lambda, \quad (10)$$

and since i as well as D is small, the curve always passes near the origin, Δ_0 having values that are sometimes positive and sometimes negative.

Thus the changing position of the magnetic axis caused by its revolution around the sun's axis of rotation produces a small shift of the displacement-curve in its own plane, together with some change of form; and from these second-order effects the inclination and period have been derived. Since the three lines selected for the investigation were observed on sixty-three days (the observations extend over an interval of one hundred and ten days), nearly two hundred separate curves enter into the calculation. Not only is the characteristic form revealed in every case, but, of far more importance as evidence of the reality of the field, the curves for the separate days are so related in form and in position with respect to the origin that the second-order quantities Y satisfy equation (9) throughout the entire series, including more than

¹ It may be added that the resulting value of i is $6^\circ.2 \pm 0^\circ.4$, while the magnetic axis revolves about the sun's axis of rotation in a period of 31.79 ± 0.31 days.

three complete revolutions of the magnetic axis about the axis of rotation.

Various details are illustrated by Figs. 3 and 4. The first gives a series of displacement curves for two dates, September 2 and 14, 1914—both the original observations and the theoretical curves—based upon the unknowns

$$x = \frac{\cos i}{k}, \quad y = \frac{\sin i \cos \lambda}{k}$$

calculated from the data for the respective days. Aside from the close accordance of the plotted points, the shift in the position of the curves with respect to the origin is to be noted. Since D was sensibly constant and equal to $+7^\circ.2$, the change in the algebraic sign of Δ for $\phi=0$ indicates a similar change in the sign of $Y=\sin i \cos \lambda$; see equation (10). The northern end of the magnetic axis was accordingly directed toward the observer on September 2 (λ near 0°) and away from him on September 14 (λ near 180°). Fig. 4 shows the close agreement of the values of $Y=y/x$ with equation (9). These results, particularly those illustrated in Fig. 4, exhibit a degree of internal consistency which is a searching test of the validity of the conclusions as to the existence of the sun's general field.

On the other hand we must consider the significance of the negative results from the lines listed in Table IV, which are not affected to any measurable amount by the sun's general field, although all of them are susceptible to the action of the magnetic fields in sun-spots. Certain aspects of the question thus raised are discussed in a later section. At present it is sufficient to state that the probabilities established by the evidence favorable to the existence of the field are so great that the negative results are presumably to be attributed to some class distinction separating the two lists of lines.

Our experience indicates that a very considerable number of lines will ultimately be found to show the influence of the sun's general field. The list of elements represented by displaced lines now includes iron, chromium, nickel, vanadium, and titanium, but it will doubtless be possible to add other elements.

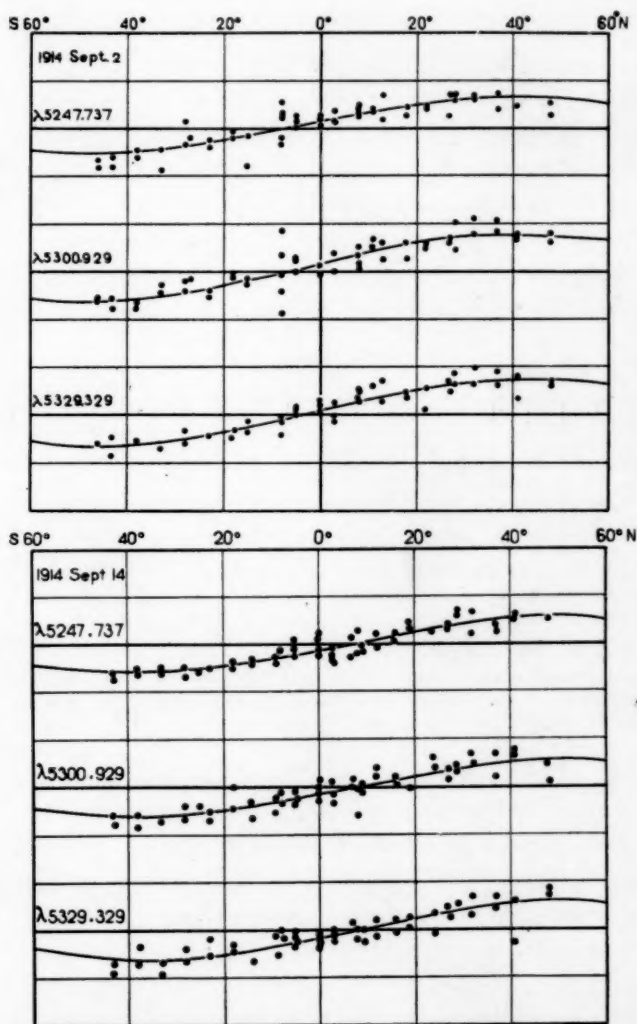


FIG. 3.—Displacement-curves for September 2 and 14, 1914. Abscissae are heliographic latitudes. Ordinates are displacements, the scale being 1 division of diagram = 0.005 mm. The curves, which correspond to equation (1), have been derived from the observed values of Δ . Their ordinates for $\phi=0$ represent the combined influence of D , i , and λ . The data for the three lines give, as mean values of $Y = \tan i \cos \lambda$, $+0.213$ for September 2 and -0.159 for September 14. These are plotted as single points in Fig. 4, together with similar values of Y for each of the other dates.

6. CALCULATION OF THE FIELD-STRENGTH

The results in Table IX are now to be combined with the laboratory data in the fourth, fifth, and sixth columns of Table X, where C , the magnetic separation for a field of one gauss, is expressed in 0.0001 A as a unit. The values of Δ_{45} in the seventh column, expressed in the same unit, depend on the sixth and last columns of Table IX. The field-strength at the magnetic pole of the sun, which, it should be repeated, is here considered to be a uniformly magnetized sphere, is then found from C and Δ_{45} by equation (5).

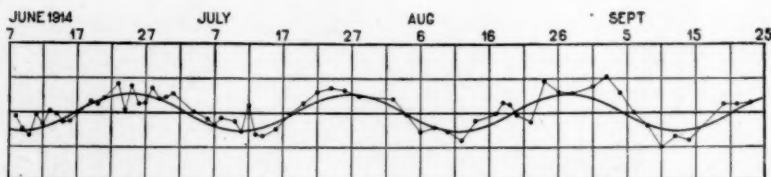


FIG. 4.—The curve $Y = \tan i \cos \lambda$. Each plotted point is derived from data of a single day similar to those illustrated in Fig. 3. Approximate values of i , P , and t_0 were read from a provisional curve. Differential corrections derived by a least-squares solution gave the final values, which correspond to the curve shown in the figure. The close agreement of the values of Y derived from the observed displacements with the theoretical curve is a most exacting test of the existence of the sun's general field.

Several of the laboratory data are from Mr. King's table¹ for iron, but most of the lines had to be specially observed. In spite of the great difficulties involved in the laboratory investigation of faint lines, Mr. Babcock has succeeded in determining C for all but four of the lines. One of these is unidentified, a second is a chromium line of intensity 0, and the others are the iron lines $\lambda 5928.013$ and $\lambda 5929.898$, both of solar intensity 2. In the previous paper doubt was expressed as to the reliability of the identification of $\lambda 5929.898$ with iron. The line has since been observed by Mr. Babcock in the spectrum of the core of the iron arc, but it does not appear in the spark and its separation by the magnet has not yet been determined. Hence we must still depend upon the approximate value of C derived from its separation in spots.

¹ *Papers Mt. Wilson Observatory*, 2, Pt. 1; *Carnegie Institution Publication*, No. 153, 1912.

TABLE X
LABORATORY DATA AND FIELD-STRENGTH

λ	El.	Int.	No. Components	$\frac{\Delta\lambda}{\mu\lambda^2}$	C	Δ_{LS}	H_p	Weight
4406.810.....	V	2	1.5	0.274	6.7	32.5	1
4418.499.....	Ti	1	6?	1.3	0.230	6.9	40.1	1
4421.733.....	V	0	12	2.4	0.45	10.1	29.9	1
4430.785.....	Fe	3	3	2.5	0.460	6.8	19.7	2
4438.006.....	V	0	4?	1.6	0.297	4.6	20.6	0.5
5247.737.....	Cr	2	3	2.54	0.653	10.1	20.6	3
5250.817.....	Fe	3	3?	1.5	0.381	5.4	18.9	2
5253.633.....	Fe	2	3?	1.5	0.377	5.4	19.1	2
5263.486.....	Fe	4	3	1.5	0.390	4.3	14.7	2
5300.929.....	Cr	2	3	1.9	0.498	6.2	16.6	2
5304.355.....	Cr	0	4	1.6	0.424	11.2	35.2	1
5328.515.....	Cr	2	9?	1.2	0.307	4.4	19.1	1
5329.329.....	Cr	3	1.7	0.440	4.5	13.6	1
5329.975.....	Cr	0	7.1
5340.639.....	Cr	0	9	1.6	0.438	8.8	26.8	2
5348.511.....	Cr	4	10?	1.6	0.429	3.2	9.9	2
5812.139.....	Fe	0	0.7	0.230	9.4	54.7	1
5828.097.....	0	8.4
5831.821.....	Ni	1	0.85	0.272	3.7	18.1	1
5856.312.....	Fe	2	1.0	0.306	9.1	39.6	1
5928.013.....	Fe	2	5.9
5929.898.....	Fe	2	0.72	9.9	18.3	1
6007.540.....	Ni	1	0.8	0.290	4.8	22.0	1
6039.953.....	V	0	4 or 6	1.5	0.502	12.1	32.2	2
6079.227.....	Fe	2	2.0	0.700	10.0	19.1	1
6111.290.....	Ni	2	3 or 4	0.9	0.315	4.2	17.8	1
6119.740.....	V	1	12	1.0	0.40	9.4	31.3	1
6129.190.....	Ni	1	1.1	0.332	7.3	29.3	1
6149.458.....	Fe	2	4?	1.3	0.446	7.4	22.2	1
6173.553.....	Fe	5	3	2.5	0.888	10.3	15.5	2

REMARKS RELATING TO LABORATORY DATA

- λ 4406 Two groups each of p - and n -components, much widened.
 4421 Weighted mean separation, based on measures of unresolved components.
 4438 Components wide, but separation well determined; two sharp p -components.
 5329.3 Only one p -component. Diffuse.
 5329.9 Very weak and diffuse. Perhaps two p -components.
 5340 Weak on laboratory plates.
 5348 Two narrow groups of n -components; probably three or four in each, unresolved.
 5831 p -components not observed; n -components not sharp.
 5856 p -components not observed.
 5928 Completely covered by an air line in spark. Inductance sufficient to cut out air line obliterates the iron line.
 5929 Not observed in spark. C from observations of sun-spots.
 6007 Difficult; p -components not observed.
 6039 Two narrow groups of n -components, unresolved.
 6079 Two n -components; violet n -component blended with adjacent line.
 6111 Very difficult; p -components not observed; two n -components.
 6119 Three groups of four components each. Measures of high weight, but C is approximate because of unequal intensities of n -components.
 6129 Difficult; p -components not observed.

The results for the structure of the lines are far from complete. In several cases the number of components is uncertain or unknown. The number of normal triplets is relatively small; nevertheless the theory developed for lines of this class or, more generally, for those with three groups of components of definite intensity relations,¹ has been applied to all of the lines observed. The error thus involved in cases of unusual structure is probably small, for the field-strength is based upon the displacement at $\phi = 45^\circ$, where the inclination of the line of sight to the lines of force is so small that the p -components and one group of the n -components are not transmitted to any appreciable extent by the analyzing apparatus of the spectrograph. Consequently the displacements must be nearly independent of the structure of the lines.² But Δ_{45} is not alone the result of observations at $\phi = 45^\circ$, for it includes displacements observed throughout the interval $\phi = 20^\circ$ to $\phi = 69^\circ$. The substantial agreement of values of Δ_{45} found from different latitudes leads, however, to the conclusion that the influence of complex structure is unimportant.

The weights of H_p in the last column of Table X have been assigned after a consideration of various circumstances that affect the precision. Both faint and strong lines are difficult to measure. Thus far it has not been found possible to use lines fainter than 0, while the large scale of the photographs gives to lines of intensity 4 or 5 a width that greatly increases the difficulty of the settings. Lines of intensity 2 or 3 are perhaps most easily measured, but, independently of intensity, contrast and sharpness enter to such a degree that general statements must be cautiously made. Further, the number of observations naturally influences the precision of Δ_{45} , the reliability in this respect being indicated by the data in Table IX. On the laboratory side there is also a wide range in precision, according to the number, character, and the more or less perfect resolution of the components of the individual lines. The adopted weights represent an attempt to strike a balance between these different factors.

It will be noted that the value of H_p for λ 5929.898 (18.3 gauss) differs widely from that of 28 gauss previously published.

¹ *Mt. Wilson Contr.*, No. 72, p. 11; *Astrophysical Journal*, 38, 109, 1913.

² *Mt. Wilson Contr.*, No. 72, p. 12; *Astrophysical Journal*, 38, 110, 1913.

The discrepancy arises from the large systematic difference between the measures of Miss Lasby and van Maanen. The original value was based upon the mean of the two series of measures, while, to secure homogeneity with the other results, that of Table X depends on the measures of van Maanen alone.

We desire at this point to express our obligations to Miss Richmond, Miss Wolfe, and Miss Felker of the Computing Division for their assistance with the least-squares reductions and the extensive calculations necessary for the determination of the field-strength from the individual lines.

7. VARIATION OF FIELD-STRENGTH WITH INTENSITY OF THE LINES

From an examination of Table X it is evident that the values of H_p vary with the intensity of the lines observed. To exhibit the nature of this relation, the data have been collected in Table XI, which also gives the weighted mean field-strength (the weights are in parentheses) corresponding to lines of each intensity for each of the elements thus far observed. It will be noted that, in general, the values of H_p corresponding to a given intensity, at least for any single element, are approximately equal. In the case of iron, intensity 2, for example, the agreement is surprisingly good, with the exception of the result for $\lambda 5856$. The mean results are shown graphically in Fig. 5.

In forming the means, the iron lines $\lambda 6079$ and $\lambda 6149$, which are slightly enhanced, perhaps should have been omitted, since lines of this character probably correspond to a higher level than that indicated by unenhanced lines of the same intensity. The mean field-strength would not, however, have been appreciably modified. The single titanium line $\lambda 4418$ is also enhanced, though not to an important degree.

For iron and chromium we find a rapid decrease in field-strength with increasing line-intensity. For vanadium, nickel, and titanium the data are too slender to establish independently the nature of the relation; but, including lines of all intensities, the mean value of H_p for each of these elements is in general agreement with the results for iron and chromium. The field-strength for iron,

TABLE XI
FIELD-STRENGTH, INTENSITY, AND LEVEL

Element	Intensity					
	0	1	2	3	4	5
Fe.....	{ 5812 54.7 (1)	5253 19.1 (2) 5850 39.6 (1) 5920 18.3 (1) 6079 10.1 (1) 6149 22.2 (1)	4430 10.7 (2) 5250 18.9 (2)	5203 14.7 (2)	6173 15.5 (2)
Means.....	5812 54.7 (1)	5753 22.9 (6)	4840 19.3 (4)	5203 14.7 (2)	6173 15.5 (2)
Level.....	270	335	370	410	450
Adopted.....	253	342	387	410	405
Cr.....	{ 5304 35.2 (1) 5340 26.8 (2)	5247 20.6 (3) 5300 16.6 (2) 5328 19.1 (1) 5278 19.0 (6)	5320 13.6 (1)	5348 9.9 (2)
Means.....	5328 26.6 (3)	5368	5320 13.6 (1)	5348 9.9 (2)
Level.....	335	368	390	418
Adopted.....	335	368	390	418
Ni.....	{ 5831 18.1 (1) 6007 22.0 (1) 6120 29.3 (1) 5989 23.1 (3)	6111 17.8 (1)
Means.....	5961	6111 17.8 (1)
Level.....	361	375
Adopted.....	331	338
V.....	{ 4421 20.9 (1) 4438 20.6 (0.5) 6030 32.2 (2)	6119 31.3 (1)	4406 32.5 (1)
Means.....	5350 29.9 (3.5)	6119 31.3 (1)	4406 32.5 (1)
Level.....	365	390	418
Adopted.....	315	317	387
Ti.....	4418 40.1 (1)
Level.....	360
Adopted.....	361

line-intensity 0, is large in comparison with the corresponding value for chromium and may be considerably in error, for it depends upon a single line, difficult of measurement on the solar plates and troublesome to investigate in the laboratory because of its faintness.

In a series of important papers¹ Mr. St. John has shown in a convincing way that lines of increasing intensity represent successively higher levels in the solar atmosphere. In accordance with

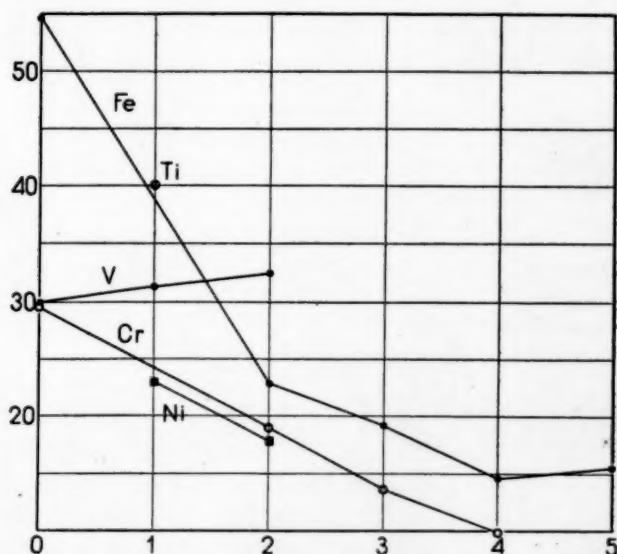


FIG. 5.—Variation of field-strength (ordinates, in gauss) with the Rowland intensity (abscissae) of lines in the solar spectrum.

the suggestion on an earlier page, the data for field-strength and intensity in Table XI and Fig. 5 are therefore open to interpretation as the result of a variation of field-strength with level, and would indicate that the intensity of the sun's general field decreases rapidly in passing upward from the level represented by lines of intensity 0.

In the main, the curves of Fig. 5 are in agreement with Mr. St. John's measures of radial velocities in sun-spot vortices. Thus

¹ *Mt. Wilson Contr.*, Nos. 69, 74, 88; *Astrophysical Journal*, 37, 322, 38, 341, 1913; 40, 356, 1914.

we find from his chart¹ that lines of iron, chromium, vanadium, and nickel having the same intensity occur at approximately the same level, while titanium lines are relatively higher by one unit of intensity. In other words, we should expect the curves to lie close together, as they actually do. The systematic difference for iron and chromium is probably real. The lines observed may actually occur at slightly different elevations, as the two curves would indicate, for we are dealing with a limited number of lines observed over the general solar disk, while St. John's results are averages for large numbers observed at the outer edge of the penumbrae of sun-spots.

But, as already stated, the correlation of the observed changes in field-strength with variations in level depends upon the assumption that the measured displacements are free from systematic errors which vary with the intensity of the lines observed. It is therefore important that the question of systematic errors be given careful attention.

8. SYSTEMATIC ERRORS

Comparing the results of different observers, the displacements determined by Miss Lasby² appear to be 50 per cent larger than those by van Maanen, and seem also to have been in excess of those found by Mr. Adams in his trial measures.³ On the other hand the mean values for Miss Richmond and Miss Felker in Table I are appreciably smaller than those by van Maanen. Further, disregarding the inconclusive discordance for $\lambda 5928$, the measures by van Maanen are in satisfactory agreement with those from the Koch registering photometer, although the data are insufficient for final conclusions.

Personal errors, however, we may suppose to be approximately constant, and should not therefore affect the relative values of the field-strength derived from lines of different intensities. Depending upon the measures of a single observer, the data discussed in this paper should be homogeneous, and it is highly improbable that the

¹ *Mt. Wilson Contr.*, No. 74, p. 5; *Astrophysical Journal*, **38**, 345, 1913.

² *Mt. Wilson Contr.*, No. 71, p. 63; *Astrophysical Journal*, **38**, 87, 1913.

³ *Mt. Wilson Contr.*, No. 71, p. 32; *Astrophysical Journal*, **38**, 56, 1913.

important relation between field-strength and line-intensity should be a consequence of personal error.

Approaching the question of systematic errors from a different direction, we are confronted by the following apparently contradictory results:

1. The three lines $\lambda 5329.329$ (Cr, 3), $\lambda 5812.139$ (Fe, 0), and $\lambda 5828.097$ (—, 0) show displacements in the second order which are sensibly equal, when reduced to the same scale, to those observed in the third order (van Maanen estimates the intensity of $\lambda 5329$ on Mount Wilson plates to be 2).

2. In third-order spectra the lines $\lambda 5247.737$ (Cr, 2) and $\lambda 5929.898$ (Fe, 2) both show displacements attributable to the sun's general field, although neither is appreciably displaced in second-order spectra (van Maanen's estimate of the intensity of $\lambda 5247$ on Mount Wilson plates is 3).

3. The lines $\lambda 6173.553$ (Fe, 5) and $\lambda 6302.709$ (Fe, 5) have wide separations of nearly equal magnitudes in the spectra of sun-spots. The former shows one of the largest displacements thus far observed in the case of the sun's general field (the measures are of third-order spectra); but the latter, $\lambda 6302$, has no measurable displacement in the first, second, or third orders.¹ (On Mount Wilson photographs $\lambda 6302$ is decidedly stronger than $\lambda 6173$ and much more difficult of measurement.)

It is difficult to find in these results any influence certainly due to intensity. The fact that the intensities of the lines under (1) are less than those under (2) might be suspected of having something to do with the failure of $\lambda 5247$ and $\lambda 5929$ to show displacements in the second order. But we should then be at a loss to know the bearing of this inference upon the third-order results for lines of high intensity, such as $\lambda 6173$ and $\lambda 6302$, referred to under (3); and the difficulty would only be increased when we

¹ This statement refers to the mean result from a number of photographs extending over a wide range of heliographic latitude. Small displacements are shown by individual groups of spectrum strips which are probably real in part, although perhaps not the result of a magnetic field. Small irregularities of photographic density may produce appreciable shifts, but these will be accidentally distributed, and have nothing to do with the systematic displacements produced by the general field. Note in this connection the results for the registering photometer given on p. 211.

attempted to account for the behavior of the faint line $\lambda 6012.450$ (Ni, 1), which has a very large displacement in the spectra of sun-spots and none at all in the general-field photographs. Clearly something besides line-intensity is involved in these phenomena.

There are reasons for believing that the amount and distribution of the photographic density across a line are factors of importance in the measurement of minute shifts.¹ Both the lines which are undisplaced in the second order have sharp edges and show strong contrast on the photographs taken in this order. The third-order images of these lines, and both second- and third-order images of the lines mentioned under (1), are much flatter. Their density-curves have neither the steepness nor the height that characterize the second-order photographs of $\lambda 5247$ and $\lambda 5929$; and, in general, lines that show displacements do not possess intensity-curves of this type. The minimum density within a line (the reference is to the negative) and the contrast vary with exposure and development. Very faint lines naturally give difficulty in measurement, but low density and excessive contrast also produce unsatisfactory results. The ease of setting is of course greater in the case of sharp narrow lines, but for such lines displacements have not been detected.

These facts may perhaps be explained by supposing that it is the point of minimum density—the highest point of the intensity-curve—rather than the edges of the line that determines the value of a setting. In cases of excessive contrast the middle of the line is likely to be underexposed and it will then be impossible to locate the minimum with precision, for throughout the central section there will be no appreciable deposit of silver.

The explanation also applies to the peculiar behavior of $\lambda 6302$ described under (3). On the Mount Wilson photographs its intensity is at least two units greater than that of $\lambda 6173$. In the third order it is so broad and diffuse that satisfactory measurement is impossible, while in the first and second orders the contrast and density are such that the point of minimum density cannot be located; $\lambda 6173$, on the other hand, is well suited to measurement.

¹ In addition to the accompanying discussion, see also Hale, *Mt. Wilson Contr.*, No. 71, p. 50; *Astrophysical Journal*, 38, 74, 1913.

It seems likely, therefore, that the form of the intensity-curve rather than intensity in the usual sense is a controlling factor in the detection and measurement of the displacements. This conclusion is not directly an answer to the inquiry as to the existence of systematic errors depending upon intensity, but it seems to account for the irregularities that might have been attributed to such an influence, and we are thus left without any evidence which would indicate that such errors have entered into the results.¹ Tentatively, we are therefore disposed to accept the relation between field-strength and intensity illustrated by the curves of Fig. 5 as an indication of a decrease in the field with increasing level in the solar atmosphere.

9. DISCUSSION OF THE RESULTS IN RELATION TO LEVEL IN THE SOLAR ATMOSPHERE

To examine further the results in relation to elevation in the solar atmosphere, they have been compared with the flash-spectrum measures of Mitchell.² A number of the lines employed were observed by him in the chromosphere; but in the nature of the case the precision of the calculated levels is less than that required to make the comparison effective, and we are therefore obliged to use mean levels. To this end the results calculated from the length of the chromospheric arcs were taken from Mitchell's tables for individual lines and grouped into means for each intensity unit of Rowland's scale. All blends and bracketed wave-lengths were omitted, and to avoid photographic influences only those lines between λ 4000 and λ 5000 were included.

This limitation is necessary in order that the calculated levels for the different elements may be comparable. The distribution of the lines throughout the spectrum is different for different elements, and, since the sensitiveness of the photographic equipment changes with the wave-length, systematic differences between the different intensity-and-level curves are likely to enter unless the

¹ A method of testing this question by measurements in the second and third orders of lines slightly displaced by the weak fields surrounding sun-spots will be tried in the future.

² *Publications McCormick Observatory*, 2, Pt. 2; *Astrophysical Journal*, 38, 407, 1913.

data relate to the same limited region of the spectrum. An exception, however, was made in the case of nickel, for which the region $\lambda 5000$ – $\lambda 6000$ was also used in order to compensate for irregularities in the interval $\lambda 4000$ – $\lambda 5000$.

The mean levels thus derived (in kilometers) have been entered in Table XI. Those for iron, chromium, and nickel show a regular increase in elevation with decreasing field-strength. For vanadium and titanium the results are discordant, but the evidence is inconclusive. Although the levels for these are systematically higher than for the corresponding intensities of other elements,

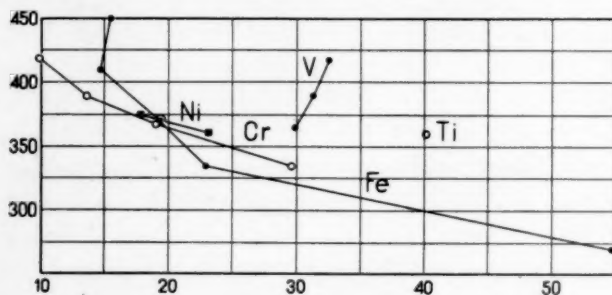


FIG. 6.—Variation of field-strength (abscissae, in gauss) with elevation in the solar atmosphere (ordinates, in kilometers). The levels require correction as described in sec. 9.

the extreme range, including all elements, is small, and apparently that portion of the sun's general field effective in producing the observed displacements is limited to a very thin shell in the solar atmosphere. The results are illustrated in Fig. 6.

Various reservations and exceptions to these conclusions immediately suggest themselves and have now to be considered.

First, we have tacitly assumed that lines of a given element and intensity correspond to the same level in the sun's atmosphere independently of wave-length. But we have reason to believe that the levels determined from line-intensity are affected by scattering,¹ which certainly varies with the wave-length; in addition there may be influences depending on wave-length which are peculiar to the different elements. We can remove the effect of scattering by the

¹ St. John, *Mt. Wilson Contr.*, No. 69, p. 17; *Astrophysical Journal*, **37**, 338, 1913.

application of an appropriate correction; but possible disturbances arising from other causes will be so involved with various uncertainties inherent in the observation of flash spectra that their consequences are necessarily disregarded.

Second, elevations derived from flash spectra must be accepted with caution, owing to the fact that for different elements lines having the same solar intensity may behave quite differently in the chromosphere, some showing much greater intensities in the flash spectrum than others. Since lines of high intensity are represented in such spectra by relatively long arcs, the calculated levels may exhibit differences that are wholly fictitious. Such, for example, is the case with vanadium as compared with the other elements observed.

Third, since the evidence for relatively greater elevation in the case of the enhanced lines seems unquestioned, it is not permissible to disregard this fact in the combination of the data now under discussion.

For a change in wave-length of 1000 Å the influence of scattering is equivalent to about one unit of intensity,¹ lines to the red corresponding to a relatively lower level than their observed solar intensities would indicate. Levels assigned on the basis of the observed intensity will therefore be too high in the red and too low in the blue. The corrections apparently should vary directly as the fourth power of the wave-length, and may be derived with the aid of the levels in Table XI, care being taken to use for each element the rate of change in elevation per unit of intensity corresponding to the element and intensity in question.

As a matter of convenience we reduce the elevations to λ 5300, the wave-length of the chromium lines. The corrections in kilometers are given in Table XII.

To study the consequences of abnormal behavior of any element in the chromosphere, we may compare the intensities of its lines in the flash spectrum with those of the flash-spectrum lines of other elements. Flash intensities do not ordinarily correspond to those of the general solar spectrum. Lines of faint solar intensity commonly have flash intensities which are one or more units higher than

¹ St. John, *loc. cit.*

the Rowland values, while the stronger lines of the general solar spectrum have relatively low intensities in the chromosphere.

TABLE XII
CORRECTIONS TO LEVEL DEPENDING ON WAVE-LENGTH

Element	Intensity					
	0	1	2	3	4	5
Fe....	5812, -17	5753, -18	4840, +17	5263, 0	6173, -45
Ni....	5989, -17	6111, -21
V....	5350, 0	6119, -28	4406, +17
Ti....	4418, +21

For the intensities involved in this investigation the behavior of iron and chromium in the chromosphere is substantially the same, and the levels calculated for their respective lines should be comparable, in so far as the point now under discussion is concerned. But with nickel, vanadium, and titanium there are deviations to be considered, which are indicated by Table XIII. For example,

TABLE XIII
FLASH INTENSITIES

SOLAR INTENSITY	FLASH INTENSITY				DEVIATIONS FROM Fe AND Cr		
	Fe, Cr	Ni	V	Ti	Ni	V	Ti
0.....	0.4	0.6	0.9	0.5	-0.2	-0.5	-0.1
1.....	0.7	1.1	1.4	1.0	-0.4	-0.7	-0.3
2.....	1.0	1.5	2.1	1.5	-0.5	-1.1	-0.5
3.....	1.4	1.4	3.0	1.8	0.0	-1.6	-0.4
4.....	2.0	1.3	4.0	2.2	+0.7	-2.0	-0.2

the relatively high flash intensities of vanadium as compared with iron and chromium lines of the same solar intensity will lead to calculated levels that are too high unless suitable corrections are applied. To derive such corrections, the flash-spectrum intensities may be plotted against the levels given in Table XI under the corresponding solar intensities. The changes in level corresponding to the differences of flash intensity in the last three columns of

Table XIII can at once be read from the curves thus defined. The results thus found are given in Table XIV.

TABLE XIV
CORRECTIONS DEPENDING ON ABNORMAL FLASH INTENSITY

Element	Solar Intensity		
	0	1	2
Ni.....	-13	-16
V.....	-50	-45	-48
Ti.....	-20

Finally, to arrive at a rough correction of the levels of the enhanced lines $\lambda 6079$ and $\lambda 6149$ which will reduce them to the system of the other lines, we may make use of the results of Mr. Adams on the displacement of lines at the sun's limb.¹ These show that the enhanced lines are shifted by larger amounts than unenhanced lines of the same intensity. For $\lambda 6079$ and $\lambda 6149$ he finds shifts of $+0.009$ and $+0.010$ mm, respectively. These are equal to the displacements of unenhanced lines whose intensities are at least two units greater than those of the lines in question. We accordingly find corrections of $+75$ km. The result is not very reliable, but the scanty data available suggest that the corrections should be larger rather than smaller. The corresponding effect upon the level for iron lines of intensity 2 given in Table XI is $+25$ km.

The application of this and the corrections in Tables XII and XIV to the levels in Table XI gives the values which are provisionally adopted as corresponding to the mean field-strengths. These results also appear in Table XI opposite the word "Adopted."

With this revision of the levels, the curves of Fig. 6 assume the form shown in Fig. 7. Notwithstanding the tentative character of several of the corrections, an improved agreement is noticeable in various directions. The iron and chromium curves are now nearly coincident, and four of the five vanadium lines, which before were seriously discordant, are now in excellent agreement with iron and chromium. The fifth vanadium line, $\lambda 4406$ (2), and the single

¹ *Mt. Wilson Contr.*, No. 43; *Astrophysical Journal*, 31, 30, 1910.

titanium line, $\lambda 4418$ (1), remain discordant, and the agreement for nickel is less satisfactory than before. The titanium line is slightly enhanced and presumably something should be added to the adopted level, but the data necessary for the calculation of the correction are lacking. It is not surprising, however, that individual lines should have levels which differ from the mean level of all lines of the same intensity; and the uncertainty of the corrections which reduce the results for nickel to $\lambda 5300$ is more than sufficient to account for the systematic deviation shown by that element in Fig. 7.

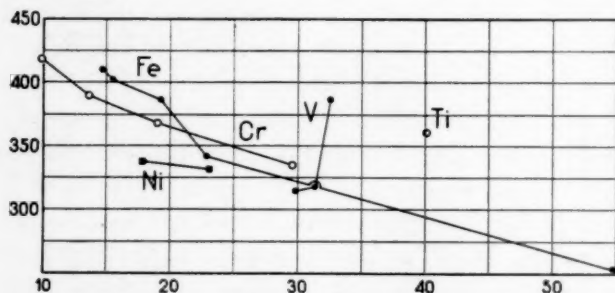


FIG. 7.—Adopted variation of field-strength with level in the sun's atmosphere. The elevations (ordinates) are in kilometers and the field-strengths (abscissae) in gauss.

The main point of this detailed discussion of corrections is that none of the disturbing influences mentioned is capable of modifying essentially what seems to be a fundamental relation between field-strength and level in the sun's atmosphere. Qualitatively the results of Fig. 7 are the same as those of Fig. 6, and quantitatively, even, the differences are confined within narrow limits.

10. INTERNAL EVIDENCE

It is now possible to point out certain internal agreements in the results that add much to the weight of the conclusions.

First, there is a wide range in the Zeeman separation of the components of the various lines under the influence of a constant field. Nevertheless, the measured displacements lead to closely accordant values for the field. As an example, the iron lines $\lambda 5253$ and $\lambda 6149$, of intensity 2, coefficient of separation about 0.4, give for H , the same value as that derived from $\lambda 5929$ and $\lambda 6079$,

also of intensity 2, whose coefficient is 0.7. In fact the ten lines of intensity 2 may be divided into two groups according to large and small values of C , respectively, with mean results as follows:

C	H_p	Weight	No. of Lines
0.676	19.8	5	3
0.378	22.5	9	7

The two groups of lines give nearly the same value for H_p , although for a given field-strength the separations of one group as observed in the laboratory are nearly twice those of the other. Their solar displacements must therefore bear a similar relation to each other, and the precision of the measures has accordingly been such as to reflect these differences in the behavior of individual lines.

Second, reference has been made to the systematic difference in field-strength for iron and chromium lines of the same intensity illustrated in Fig. 5. The difference does not appear in Fig. 7, and upon examination we find that this is due to the circumstance that the calculated levels for chromium are higher than those for iron (see Table XI), the differences in field and intensity compensating each other in such a way that for a given level the calculated strength is the same whether we use the lines of iron or chromium. In other words the hypothesis of a change of field with level reconciles the differences of Fig. 5, and the observed displacements for these two elements differ by just the amount required for them to yield the same field-strength for a given elevation.

Third, the application of the corrections derived in the preceding section led in general to an improved agreement. Since all of the corrections are based upon the hypothesis of levels, the accordance of the final results is in favor of the applicability of this hypothesis.

II. FAILURE OF CERTAIN LINES TO REVEAL THE GENERAL FIELD

The influence of the density and contrast of a spectral line upon the detection of displacements due to the sun's general field has already been discussed. The apparently great importance of these factors raises a question as to the part they may have played in producing the negative results summarized in Table IV. It is certain, however, that the character of these lines is not wholly responsible for the absence of measurable displacements, and it is unlikely

that it has had any considerable influence, for many of the lines are of a satisfactory quality.

It is evident, therefore, that in the case of Table IV we are dealing with an anomaly of another kind. All of the lines there listed show displacements in the spectra of sun-spots. For some of these lines the separation of the components in spots is less than the average spot-separation of the lines in Table I, but the behavior of others (for example, λ 6005, λ 6012, and λ 6081) in spots is entirely comparable with that of the lines in Table III. The line λ 6012 (Ni, 1) is especially noteworthy. Its components are more widely separated by the magnetic fields in sun-spots than those of any of the lines in Table III excepting λ 6173 (Fe, 5); but on the general-field plates it shows no appreciable displacement, although other lines of similar intensity on the same photographs, such as λ 6007 and λ 6039, are certainly displaced.

There is obviously a lack of parallelism between the results for spot fields and for the sun's general field. At present no complete explanation of these differences can be given; we can only offer various suggestions that later experience may prove to be of more or less significance.

In general it is to be noted that sun-spot fields extend over a much greater range of level than can at present be explored in the case of the sun's general field. Moreover, the conditions of pressure, temperature, and ionization are more or less different in the two cases; and it is not surprising that complete parallelism in results should not be found for fields existing under such diverse conditions. It seems not improbable that, in the sun's general atmosphere, some of the lines listed in Table IV originate outside the thin shell of a hundred or more kilometers' thickness which seems to include that part of the field now accessible to observation.

A careful examination of the displacements in sun-spot vortices by Mr. St. John and Miss Ware reveals no essential difference between the two classes of lines. On the other hand, Mr. King, from a consideration of temperature conditions, finds for chromium and vanadium, at least, some indication of a class distinction. His conclusions are as follows:

The iron and nickel lines of Table III are faint in the arc and for the most part have not been observed in the furnace. The chromium and vanadium lines are usually strong at all furnace temperatures. The vanadium lines λ 4438 and λ 6120 are relatively strong at the lowest temperature at which the spectrum appears.

Table IV contains no lines which are in any respect low-temperature lines. The iron lines are of the type seeming to require not only high temperature but high-vapor density. They are shown best by the core of the arc and by the tube-arc, and with great difficulty by the furnace and the spark. Two titanium lines and one calcium line are produced by the furnace, but are faint or absent at low temperature.

12. THE LOCAL-WHIRL HYPOTHESIS

In *Mount Wilson Contribution*, No. 71, it was suggested that the general magnetic field of the sun might result from the combined effect of a great number of local whirls. Evidence opposing this view was offered, but the material available for discussion was insufficient, and we may now return to a consideration of the suggestion, which is regarded favorably by both Birkeland¹ and Brunt.²

The hypothesis assumes the existence of a large number of minute spots or pores, too small to be distinguished as such, but having magnetic and other properties similar to those of visible spots. Recent investigations at Mount Wilson show these properties to include:

1. An almost universal tendency of spots to occur in pairs, the chief members of which are of opposite magnetic polarity. This tendency to form bipolar groups was shared by the few extremely small spots which appeared near the last sun-spot minimum, when the general magnetic field was under observation.

2. The preceding members of bipolar spot-groups, in the great majority of cases, are of opposite polarity in the northern and southern hemispheres.

3. Since the last sun-spot minimum the preceding members of bipolar groups have been opposite in polarity to those observed in the same hemisphere before the minimum.

¹ *Comptes Rendus*, 157, 394, 1913.

² *Astronomische Nachrichten*, 196, 169, 1913.

4. The magnetic axes of sun-spots are in approximate coincidence with solar radii.

5. The strength of the magnetic field of a spot is roughly proportional to its area.

Minute bipolar spots, even if distributed uniformly over the sun, could not account for the general field, for the following reasons:

1. If the members of each bipolar group were of equal field-strength, they would exactly neutralize each other in their combined effect.

2. If, as is usually the case, the preceding members of bipolar groups were larger, on the average, than the following members, the polarity of a general field due to their predominating magnetic influence should have reversed at the sun-spot minimum. Our observations show, on the contrary, that the polarity of the general field did not reverse at the minimum.

Suppose we assume, however, that the minute dark spaces between the brighter granules of the solar surface represent small single spots, opposite in polarity in the northern and southern hemispheres. It is not clear why they should fail to show the bipolar characteristics of larger spots, but, as single spots with almost no indications of bipolar structure are not very uncommon, we may make this assumption. It is more difficult to admit the possibility that their polarity did not reverse at the sun-spot minimum, because single spots, as well as bipolar groups, shared in this remarkable change. In any case, however, the hypothesis must face the fact that the magnetic axes of visible spots are very nearly radial, and that the granulated structure is best seen at the center of the sun, as though we were looking down through the bright granules into the darker regions between them. Under such conditions it is hard to see how we could fail to observe the Zeeman effect to the best advantage (along the lines of force) at the center of the sun, where the general magnetic field shows no displacements. If this objection were waived, it would still be necessary to assume a distribution of the granules such as to account for the observed inclination of the magnetic axis to the axis of rotation ($6^{\circ}2$). Finally, there would remain the serious difficulty, pointed out by Brunt, of supposing that the total charge

of electricity in a pore, or in a minute vortex between the granules, is almost as great as that required to account for the intense magnetic field in a large sun-spot.

While for the foregoing reasons we are not inclined to give favorable consideration to the hypothesis of local whirls, further attempts will be made to overcome the observational difficulties of photographing the spectra of the dark spaces between the granules on a sufficient scale to detect possible local Zeeman effects.

In considering the bearing of our results on theories of the solar magnetism it should be remembered, quite apart from the question of systematic errors, that our highest values do not necessarily represent the full intensity of the sun's field. On the contrary, the apparent variation of the field-strength with level in the solar atmosphere renders it probable that more intense fields may ultimately be detected at lower levels. This change of field-strength, if actually as rapid as the results seem to indicate, will also afford a useful criterion of a satisfactory theory, which must furthermore account for the observed inclination of the magnetic axis to the sun's axis of rotation.

13. SUMMARY

The present investigation is a continuation of that in *Contribution*, No. 71. Measures of displacements are given for twenty-six additional lines in the solar spectrum belonging to the elements iron, chromium, nickel, vanadium, and titanium (Tables V-VIII). Eighteen other lines (Table IV), all of them susceptible to the influence of magnetic fields in sun-spots, show no measurable shift. The twenty-six lines, which through changes in position indicate the presence of a magnetic field (Table IX), confirm the results detailed in *Contribution*, No. 71, and seem to place beyond reasonable doubt the conclusion that the sun behaves approximately as a uniformly magnetized sphere (secs. 1, 2, and 5), with the magnetic axis only slightly inclined to the solar axis of rotation and a polarity corresponding to that of the earth.

Laboratory data are available for twenty-seven of the thirty lines (indirectly, for one line) known to be influenced by the sun's general field. Combined with displacements observed in the solar

spectrum, these yield for each line a value of the field-strength at the magnetic pole (sec. 6 and Table X). An intercomparison of results (Table XI) shows that the field decreases with increasing values of the Rowland intensity of the respective lines (sec. 7). Since line-intensity increases with the level at which the various lines originate, it would appear that the strength of the sun's general field falls off rapidly with increasing elevation in the solar atmosphere.

Using Mitchell's observations of the flash spectrum, we find that the part of the field now accessible to observation lies within the bounding surfaces of a thin shell in the solar atmosphere, whose thickness seems to be of the order of 150 km (Table XI, Fig. 6). The application of certain necessary corrections (Tables XII and XIV) improves the internal agreement and leaves outstanding as discordant only two of the twenty-seven lines (Fig. 7). With proper allowance for peculiarities in the behavior of different elements and in the Zeeman effect for individual lines, the results indicate that definite values of the calculated field-strength always correspond to definite levels in the solar atmosphere.

Systematic errors in the measured displacements varying in an appropriate manner with the intensity of the spectral lines would explain the observed dependence of field-strength upon line intensity; but there is no indication of the existence of such errors (sec. 8), and the internal evidence, on the other hand, is strongly in favor of the hypothesis of changing field-strength with changing level (sec. 10).

The anomalous behavior of the lines which show no displacement is not satisfactorily explained. Because of possible peculiarities of pressure, temperature, and electrical conditions necessary for the emission of these lines, they may originate outside the very limited region within which it has been possible thus far to observe the sun's general field (sec. 11).

The underlying causes of the sun's general magnetic field remain obscure. The evidence bearing on the local-whirl hypothesis is unfavorable to its acceptance as an adequate explanation of the existence of the field (sec. 12).

MOUNT WILSON SOLAR OBSERVATORY
February 1918

ON PARALLAXES AND MOTION OF THE BRIGHTER
GALACTIC HELIUM STARS BETWEEN GALACTIC
LONGITUDES 150° AND 216° —*Concluded*

By J. C. KAPTEYN²

21. LUMINOSITY-CURVE OF THE Bo-B5 STARS

In accordance with earlier results, I will assume that the luminosity-curves are Gaussian error-curves, with the equation

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2(M-k)^2}. \quad (100)$$

Care will, of course, be taken to note any indicated deviations from this form, particularly for the fainter half of the curve, for which we have scarcely any data from direct observations.

First solution: from the Cannon stars in the Nebula-group.—Since it was assumed that the number of Bo-B5 stars fainter than 9.05 necessary to complete the quantities in the third column of Table XXX probably lies between 0 and 6, I will assume for this number the value 3. Any error thus committed will be of little importance. The arithmetic mean of the magnitudes thus becomes 6.58, and the median magnitude 6.48. If the luminosity-curve is an error-curve, or even a symmetrical curve, both the arithmetic mean and the median will coincide with the maximum. I thus adopt for the maximum of the frequency-curve of the apparent magnitudes $m_{\max} = 6.53$. As one-twelfth of the stars in this region belong to the outside stars (see Section 18) whose maximum (arithmetic mean) lies at $m = 6.06$, we have

$$\text{Corrected } m_{\max} = 6.57. \quad (101)$$

For the determination of the probable amount of deviation (probable error) we have from the seventh column of Table XXX

$\frac{1}{4}$ of the stars are fainter than 7.90. Therefore $r = 7.90 - 6.57 = 1.33$

$\frac{1}{4}$ of the stars are brighter than 5.78. Therefore $r = 6.57 - 5.78 = 0.79$,

¹ *Contributions from the Mount Wilson Solar Observatory*, No. 147.

² Research Associate of the Carnegie Institution of Washington, Mount Wilson Solar Observatory.

so that in the mean we have $r = 1.06$. Further, the parallax of the group being $0''.0054$,

$$M = m - 6.34. \quad (102)$$

We thus have for the constants of the luminosity-curve (100)

$$\left. \begin{aligned} K &= 6.57 - 6.34 = +0.23 \\ h &= \frac{0.4769}{r} = 0.450 \end{aligned} \right\} \quad (103)$$

Second solution: from the outside Cannon stars.—Treating the numbers of the eighth column of Table XXX in the same way, we find, as remarked in Section 18, a value of K which must be substantially correct and a value of h which must be too low. Noting that for the mean parallax $0''.0081$ of the outside stars

$$M = m - 5.46 \quad (104)$$

we find (27 stars)

$$K = +0.60 \quad h > 0.40 \quad r < 1.19. \quad (105)$$

Third solution: from the bright stars in Table XXXIX—Arranging the stars for which $\lambda < 165^\circ$ according to absolute magnitude, M , and parallax, we obtain the summary¹ given in Table XXXI.

Treating the numbers in Table XXXI exactly as was done in the similar case of *Mount Wilson Contribution* No. 82, pp. 41 and 42, I find for the luminosity-curve the results shown in Table XXXII.

The third column of Table XXXII gives the numbers corresponding to the observed luminosity-curve, the number of stars on which each value rests being added in parentheses. The extreme values are thus seen to be very uncertain. It seems impossible to determine from these data the values of both K and h in (100). I therefore adopted $K = +0.36$, which is the weighted mean of the values obtained in the first and second solutions, and found

$$h = 0.50. \quad (106)$$

¹ The absolute magnitudes were obtained by using the values of $\pi_{2.5}$ instead of the finally adopted π . For the luminosity-curve the difference is immaterial.

The theoretical curve computed with these values of h and K is in the fourth column of Table XXXII. The enormous residual for $M=+0.5$ is not to be taken too seriously. It corresponds to an irregularity in the actually observed numbers of less than three stars.

TABLE XXXI
NUMBER OF B0-B5 STARS

M	\bar{M}	π									
		0°.0194	0°.0154	0°.0122	0°.0097	0°.0077	0°.0061	0°.00485	0°.00385	0°.00305	
+1.75 to +2.25	+2.0										
+1.25 " +1.75	+1.5										
+0.75 " +1.25	+1.0										
+0.25 " +0.75	+0.5										
-0.25 " +0.25	0.0										
-0.75 " -0.25	-0.5										
-1.25 " -0.75	-1.0										
-1.75 " -1.25	-1.5										
-2.25 " -1.75	-2.0										
-2.75 " -2.25	-2.5										
-3.25 " -2.75	-3.0										
-3.75 " -3.25	-3.5										
-4.25 " -3.75	-4.0										
-4.75 " -4.25	-4.5										
-5.25 " -4.75	-5.0										
Total below crosslines											

TABLE XXXII

LUMINOSITY-CURVE B0-B5 STARS

M	\bar{M}	Number	Comp.	O-C
+1.25 to +0.75	+1.0	17.0 (0.5)	17.3	-0.3
+0.75 " +0.25	+0.5	38.6 (5.5)	18.8	+19.8
+0.25 " -0.25	0.0	23.9 (10)	18.2	+5.7
-0.25 " -0.75	-0.5	10.2 (7)	15.9	-5.7
-0.75 " -1.25	-1.0	9.0 (9)	11.9	-2.9
-1.25 " -1.75	-1.5	7.0 (7)	8.1	-1.1
-1.75 " -2.25	-2.0	6.0 (6)	4.9	+1.1
-2.25 " -2.75	-2.5	2.0 (2)	2.4	-0.4
-2.75 " -3.25	-3.0	3.0 (3)	1.2	+1.8
-3.25 " -3.75	-3.5	1.0 (1)		
-3.75 " -4.25	-4.0	0.0 (0)	0.7	+1.3
-4.25 " -4.75	-4.5	1.0 (1)		

Collecting these results, including the solution of *Mount Wilson Contribution*; No. 82, p. 43, with weights according to a rough estimate of the reliability, we have:

CONSTANTS LUMINOSITY-CURVE B0-B5 STARS

	No. of Stars	K	Weight	<i>h</i>	Weight
First solution.....	48.0	+0.23	1.8	0.450	1
Second ".....	27.0	+0.60	1.0	(>0.400)	0
Third ".....	69.0	0.500	1
Mt. Wilson Contr. No. 82	142.5	+0.885	1.0	0.409	2
Adopted means....	+0.500	0.442

(107)

To this value of *h* corresponds the value $r = \pm 1.01$ mag., that is, the B0-B5 stars are distributed according to an error-curve about the central value $M = +0.50$, with a probable error of about one magnitude.

22. LUMINOSITY-CURVE OF THE B8-B9 STARS

First solution: from the Cannon stars in the Nebula-group.—In Section 18 reasons were given for the assumption that, for the nebula region, the number of stars from 9.05 to 9.55 in Table XXX, in order to become complete, must be multiplied by a factor not exceeding 2. This being admitted, it is evident that the maximum of the frequency-curve of the apparent magnitudes must lie at about

$$m = 8.80. \quad (108)$$

Fitting an error-curve having this central value to the observed numbers, I find

$$h = 0.90. \quad (109)$$

The correction required for the position of the maximum on account of the admixture of stars not belonging to the group is +0.10 mag. Consequently, by (102), the elements of the luminosity-curve are

$$\left. \begin{aligned} K &= 8.80 + 0.10 - 6.34 = +2.56 \\ h &= 0.90 \end{aligned} \right\} \quad (109 \text{ stars}) \quad (110)$$

Second solution: from the outside stars.—Neglecting the stars fainter than 9.05, the maximum is located somewhere near $m = 8.30$.

The value of h found by fitting an error-curve to all data is 0.78, but, for reasons already given, this value must be too small. Hence, by (104),

$$\left. \begin{array}{l} K = 8.30 - 5.46 = +2.84 \\ h > 0.78 \end{array} \right\} \quad \begin{array}{l} (143 \text{ stars}) \\ (111) \end{array}$$

The bright B8-B9 stars are too few in number to yield any reliable result. The same holds for the stars in *Mount Wilson Contribution*, No. 82. Finally, therefore, we have from (110) and (111), giving equal weight to the two values of K ,

$$K = +2.70 \quad h = +0.90 \quad (\text{B8-B9 stars}) \quad (112)$$

23. LUMINOSITY-CURVE OF THE B₃ STARS

In passing from the B0-B5 stars to the B8-B9 stars, the change in the value of K is so considerable that it becomes highly important to attempt a still further subdivision. For the B₃ and B₅ stars the data are still fairly adequate to such a determination. For the former I use the Cannon stars, both for the nebula and the outside region and further the data contained in *Mount Wilson Contribution*, No. 82, discussed in the same way as the bright B0-B5 stars in the third solution of Section 21.

In the case of the outside stars there is no objection to the value for K . The result for h is really (see Section 18) a lower limit. However, in the cases already treated, this value differs so little from that for the nebula region that in the present instance—for which only a rough result is all that can be hoped—I have no hesitation in using the h furnished by outside stars on the assumption that they are all at the same distance. The reduction to absolute magnitude is of course made with the mean parallax (33). Table XXXIII summarizes all the data. The apparent magnitudes corresponding to the values of M in the table were obtained by (102) and (104). The numbers from *Mount Wilson Contribution*, No. 82, were multiplied by a factor such that the total brighter than $M = +1.75$ becomes equal to that for the stars in the nebula region increased by those outside. They thus become comparable with the sums of the two preceding columns. This reduction is necessary because only that part of the curve which corresponds

to the stars brighter than $M=+2.25$ is given. The sudden breaking off of the curve at this point by no means implies the non-existence of fainter stars. In parentheses are added the

TABLE XXXIII
LUMINOSITY-CURVE B₃ STARS

M	NUMBER OF STARS					
	Neb.	Outside	Contr. 82	Adopted	Comp.	O-C
$>+ 2.75$	0	0.0	0.0	1.7	-1.7
$+2.25$ to $+2.75$..	0	0.0	0.0	1.7	-1.7
$+1.75$ " $+2.25$..	3	0.0	9.0 (1)	3.0	2.7	+0.3
$+1.25$ " $+1.75$..	3	4.0	5.0 (2)	6.0	3.7	+2.3
$+0.75$ " $+1.25$..	3	0.0	5.0 (5)	4.0	4.5	-0.5
$+0.25$ " $+0.75$..	1	0.0	4.5 (9)	3.0	4.8	-1.8
-0.25 " $+0.25$..	4	2.5	3.1 (11)	4.8	4.4	+0.4
-0.75 " -0.25 ..	4	0.5	2.8 (13)	3.7	3.5	+0.2
-1.25 " -0.75 ..	0	2.0	2.7 (12.5)	2.4	2.7	-0.3
-1.75 " -1.25 ..	2	2.0	1.2 (5.5)	2.1	1.6	+0.5
-2.25 " -1.75	1.0	1.3 (6)	1.2	0.9	+0.3
-2.75 " -2.25	0.4 (2)	0.4	0.4	0.0
$<- 2.75$	0.3	-0.3

observed numbers of stars on which the results of *Mount Wilson Contribution*, No. 82, rest. They determine the weight with which these values are to be combined with the totals of the two preceding columns. The results of the combination are given in the column headed "adopted," and are fairly well represented by an error-curve for which

$$K = +0.53 \quad h = +0.52 \quad (\text{B}_3 \text{ stars}) \quad (113)$$

24. LUMINOSITY-CURVE OF THE B₅ STARS

It is seen that among the outside B₅ stars in Table XXX there is none fainter than 8.5. We may conclude, I think, that we have before us the complete frequency-curve. According to the last line but one we have for the maximum, $m_{\max} = 7.73$ (outside stars). In the nebula region two of the observed stars are fainter than 8.5, but none fainter than 9.0. It seems hardly safe to suppose that here, too, we have the whole of the frequency-curve before us. If we assume (a) that the frequency-curve is complete, (b) that there are two stars of magnitude 9.1 really existing but not observed, the

probability is that the truth lies between the two. Corresponding to supposition (a) we find $m_{\max}=7.50$, and to (b) $m_{\max}=7.69$. The difference is not very material and I adopt

$$m_{\max}=7.60 \quad (\text{neb. region}) \quad (114)$$

Transferring to absolute magnitudes by (104) and (102) we have

$$\left. \begin{array}{ll} \text{for the outside stars} & K=2.27 \quad (7 \text{ stars}) \\ \text{for the Nebula-group} & K=1.26 \quad (15 \text{ stars}) \\ \text{Weighted mean} & K=1.58 \quad (22 \text{ stars}) \end{array} \right\} \quad (115)$$

The agreement is poor. For the derivation of h I added the data of *Mount Wilson Contribution* No. 82 to those of Table XXXIII, just as was done for the B₃ stars, and found

$$h=0.50. \quad (116)$$

25. FURTHER LUMINOSITY-CURVES AND SUMMARY OF RESULTS

By the combination of the results (112) with (115) and (116) we obtain for the B₅-B₉ stars

$$K=+2.61 \quad h=0.87 \quad (274 \text{ stars}) \quad (117)$$

There is further the determination of *Mount Wilson Contribution*, No. 82, p. 45,

$$K=+2.00 \quad h=+0.508 \quad (69 \text{ stars}) \quad (118)$$

which is confessedly poor; the solution (117), too, is not of high weight, but doubtless much better. Combining with the respective weights 4 and 1, I find

$$K=+2.49 \quad h=+0.80 \quad (\text{B}_5\text{-B}_9 \text{ stars}) \quad (119)$$

For the B₀, B₀-B₂, and B₁-B₂ stars, the value of K was obtained by taking the arithmetic means of the apparent magnitudes of the Cannon stars in the nebula region and reducing to absolute magnitudes by subtracting 6.34 according to (102). Further, in the case of the B₀-B₂ stars, I derived a very crude value of h . For the B₀-B₉ stars a good result could not be obtained. All the results are summarized in Table XXXIV, in which, for convenience, I also insert the constants for the A₀-A₉ stars, which will be obtained presently.

In the fifth column have been inserted the numbers of stars on which the several determinations rest. These give a very imperfect idea of the reliability of the corresponding luminosity-curve, not only because the absolute magnitudes are wholly dependent on the accuracy of a few parallaxes, but mainly because the observations embrace very different fractions of the whole curve. For this reason the part of the curve covered by observations has been

TABLE XXXIV
CONSTANTS OF LUMINOSITY-CURVES

	K	h	r	No. of Stars	Fraction of Curve	Quality
Bo.....	-2.5	?	?	6	1.00	Poor
B3.....	+0.5	0.52	0.92	99	0.95	Good
B5.....	+1.6	0.50	0.95	22, 49*	0.81	Fair
Bo-B2.....	-1.6	0.43:	1.11:	13	1.00	Poor
Bo-B3.....	-0.4	0.52	0.92	33	1.00	Fair
B1-B2.....	-0.9	?	?	7	1.00	Poor
Bo-B5.....	+0.5	0.442	1.01	286	0.96	Good
B5-B9.....	+2.5	0.80	0.60	343	0.50	Fair
B8-B9.....	+2.7	0.90	0.53	252	0.50	Fair
Ao-A9.....	+3.4	0.80	0.60	474, 601†	0.50	Fair

* Twenty-two stars used for the derivation of K ; 49 for h .

† 474 stars used for the derivation of K ; 601 for h .

roughly indicated in the sixth column. When the fraction is less than 0.50, the maximum is not included, and its determination becomes more or less precarious. As a consequence, the elements for the stars of type B3, resting on 99 stars, must be far better than those for the B5-B9 stars, for which 343 stars were available. Even the values of K for the Bo, Bo-B2, B1-B2 stars, though poor, deserve more confidence than we might at first sight be willing to concede on account of the extremely small numbers of stars on which they rest, simply because practically the whole of the curve is covered by observation. A crude estimate of the reliability of the elements has been given in the last column.

I have computed these numerous luminosity-curves in the hope that they might help materially in obtaining good parallax estimates when other means are defective or wanting.

Remark.—There are a few exceptionally bright stars in our lists, such as ϵ and ζ Orionis and ϵ Canis Majoris.¹ For all three the absolute magnitudes are brighter than -4.0 . Since the spectrum of the last is B1, and the others are probably B1 or B0, the magnitudes do not seem irreconcilable with the luminosity-curves of Table XXXIV. The case of β Orionis (Boss 1250) is different. It is the brightest star in our lists, $M = -5.5$, whereas the spectrum is B8; hence it is more than 8 mags. brighter than the mean for stars of the same spectrum, and seems to deserve particular attention.

26. LUMINOSITY-CURVE OF THE A STARS

The value of K was determined from the outside stars. The main difficulty was the probable incompleteness of the numbers of stars between magnitude limits 9.05 and 9.55. We are not altogether without means for overcoming the difficulty, however.

We can make a rough determination of ϵ , the factor by which the observed number must be multiplied to make it complete, with the aid of the B5–B9 stars, by comparing the observed and the theoretical numbers obtained with the constants of Table XXXIV. We find

	No. Obs.	No. Theor.	ϵ
Nebula region	12	30.5	2.5
Outside stars	7	11.2	1.6
Total	19	41.7	2.2

As already remarked, the A stars in the nebula region scarcely admit of a value of ϵ as high as 2.0. The theoretical number for the nebula region, therefore, and hence also for the outside stars, must be too high. I assume

$$1.0 < \epsilon < 2.4. \quad (120)$$

If we try to represent the numbers of the last column of Table XXX by an error-curve on the two hypotheses $\epsilon = 1$ and

¹ The Boss numbers are respectively 1370, 1398, 1804.

$\epsilon = 2.4$, we find maxima at $m = 8.80$ and 9.05 , respectively, so that we cannot greatly err if we write $m = 8.93$. We thus get, by (104),

$$K = 3.47. \quad (121)$$

Adopting this value, the maximum in the Nebula-group, according to (102), must lie at $m = 9.81$, and, corrected for the large admixture of external stars projected on the group, for all the A stars within the limits (78) $m = 9.29$. From both regions together I then obtain the best representation by assuming

$$h = 0.80. \quad (122)$$

As there may still remain some doubt as to the correctness of K , particularly because the preceding determination tacitly assumes that for the region considered the parallax of the bright A stars agrees with that of the bright B₀–B₉ stars, I have tried to find some verification by entirely independent data. The most suitable for the purpose is that furnished by the Pleiades and the Hyades.

If the stars in these groups be arranged in order of magnitude, they will also be arranged nearly in the order of spectrum. This remark is not new, but Table XXXV, which brings the fact clearly into view, may not be unwelcome.

a) *The Pleiades*.—The brighter stars are all B₅–B₉. For stars of magnitude 6.59 and fainter, with relatively few exceptions down to 9.05, the spectrum is A. Stars still fainter show an F spectrum. Outside the limits 6.5 and 9.1 no A stars occur, though we should expect a few to appear were the number of stars a hundred times greater. It is evident that we have before us the entire luminosity-curve and that the absolute magnitudes of the bulk of the A stars lie within a range of 2.6 mags. If we assume that the luminosity-curve is an error-curve, its maximum must coincide with the arithmetic mean. From the photographic magnitudes determined at Potsdam I find from 23 A stars

$$m_{\max} = 7.70 \quad h = 0.82 \quad (123)$$

and for 8 B₅ stars

$$m_{\max} = 4.49. \quad (124)$$

It thus appears that the maximum of the frequency-curve for the A stars is 3.21 magnitudes fainter than that for the B₅ stars.

TABLE XXXV
DISTRIBUTION OF SPECTRA

Mag.	Hyades*	Pleiades†
3.41.....		B ₅
3.62.....	A ₅	
3.63 to 4.10	K, K, K, K.....	B ₈ , B ₅
4.10 " 4.55	A, A ₃ , A ₃ , A ₅	B ₅ , B ₅
4.55 " 5.05	A ₅ , A ₅ , A ₂ , A ₅ , A ₃	B ₅
5.05 " 5.55	A ₅ , A ₈ , A ₅ , A, A ₂ , A, A ₅ , A ₅	B ₈
5.55 " 6.05	A, A ₅ , A ₅ , G, A ₂ , A ₉ , F, A, F, A ₄ , A ₉ , F, A ₅ , A ₉	B ₅ , B ₈ , B ₈ , B ₈
6.05 " 6.55	A ₉ , A ₈ , A ₈	B ₈
6.55 " 7.05	F ₂ , F ₅ , F ₂ , G ₀	A, B ₈ , A, A, A
7.05 " 7.55	G ₀	A, A, A, A, A
7.55 " 8.05	G ₀ , G ₀ , G, G ₀ , F ₈	A, A, A, A, A ₅
8.05 " 8.55	G ₀ , G ₀ , G ₀ , G ₅	A ₂ , A ₃ , A ₅ , A ₂ , A ₂ , A ₅ , A ₅
8.55 " 9.05	G ₅ , F ₈ , K ₀ , K ₀ , K ₂ , G ₀ , K ₀ , K ₀ , G ₅	F, F, A ₅ , A
9.05 " 9.55		F ₂ , F, F ₂ , F
10.45.....		F ₅

* The stars belonging to the Hyades were taken from the preface to *Groningen Publications*, No. 23, the data for spectrum being largely supplemented by private letters from the Harvard Observatory and by plates taken at Potsdam by Dr. Zernike. The list contains all the stars of determined spectrum known to belong to the group.

† The photographic magnitudes for the Pleiades are by Hertzsprung, *Pub. Astr. Obs. Potsdam*, 22, No. 63, p. 21. It would have been preferable to use the visual magnitudes by Müller and Kempf (*Astronomische Nachrichten*, 150, 193, 1899), but I have not deemed it important to make the change. The brighter stars have been reduced to the Harvard scale by subtracting 0.24 mag. The spectra for the brighter stars are from Miss Maury (*Harvard Annals*, 28, Part I); for the fainter stars they are from Tikhoff (*Mitteilungen Pulkowo*, No. 40, Tableau II). The proper motions necessary for determining whether a star belongs to the physical group are from the following sources: Lagrula or Elkin (weight 1), Boss (weight 1), and Stratton (weight 1).

The same must also be true for the luminosity-curves, and since by (115) the maximum of the luminosity-curve for the B₅ stars lies at absolute magnitude $K = +1.58$, we find

$$\text{A stars} \quad K = 4.79, \quad (125)$$

a determination which is independent of the parallax of the Pleiades.

b) *The Hyades*.—Table XXXV contains all the stars of determined spectrum which are known to belong to the group.

We here meet with the curious fact that four of the five brightest stars have K spectra, a type not again appearing until we reach

stars fainter than 8.5. If I am not mistaken this fact first gave rise to the general¹ theory of giant and dwarf stars.

For the determination of the luminosity-curve we cannot use all the objects in Table XXXV, for it is evidently necessary to know *all* the stars belonging to the group, together with their spectra, down to a specified and rather faint limit, a condition satisfied only for that part of the group for which proper motion plates have been measured at Groningen.

As the spectra are still rather uncertain, I give in full all the A stars which, according to the data at my disposal, occur in the restricted area. It will thus be easy to improve the results as soon as better data are available.

No. <i>Gron. 23</i>	Harv. Mag.	Spectrum	No. <i>Gron. 23</i>	Harv. Mag.	Spectrum
227.....	3.62	A5	87.....	5.68	A2
143.....	4.24	A	245.....	5.70	A9
170.....	4.60	A5	64.....	5.76	A4
315.....	4.75	A5	371.....	5.80	A6
107.....	4.84	A2	222.....	5.97	A5
251.....	4.84	A5	282.....	6.04	A9
385.....	4.85	A3	44.....	6.14	A9
38.....	5.27	A5	1.....	6.35	A8
255.....	5.49	A5	150.....	6.39	A8
34.....	5.59	A5			

Among the stars fainter than 6.39 there is not a single A star.

The mean magnitude

$$m = 5.36 \quad (19 \text{ stars}) \quad (126)$$

is the apparent magnitude corresponding to the maximum of the frequency-curve. For the best-fitting modulus of precision I find

$$h = 0.88. \quad (127)$$

For the conversion into absolute magnitudes we require the parallax of the group, which according to *Groningen Publications*, No. 23, p. 4, is

$$\pi = 0''.024, \text{ therefore } M = m - 3.10. \quad (128)$$

¹ General in the sense that the theory applies, not only to certain clusters, but to the whole of the stellar system.

Finally, for the elements of the luminosity-curve

$$K = 5.36 - 3.10 = 2.26 \quad h = 0.88 \quad (129)$$

The agreement of the values for h in (129) and (123) *inter se* and with (122) is good. The agreement of (125), (129), and (121) for K is surprisingly bad.

I think that one of the causes must be an error in the value (129) furnished by the Hyades. For, adopting $K = +2.26$, we find by (102) and (104) that the maximum in the Nebula-group is at $m = 8.60$ and in the outside group at $m = 7.72$. A glance at Table XXX shows that such values are not to be thought of. After mature consideration I believe that the error must in great part be attributed to the parallax. We have two excellently agreeing values, one by Boss from the convergent of the proper motions and some measures of radial velocity; the other a direct determination of parallax made at Groningen. The following criticisms may, however, be made.

In the Groningen determination the magnitude error offered considerable difficulty and it does not seem impossible that, notwithstanding all our care, an error of $0''.01$, though four times the probable error, may have crept into the result.

In Boss's determination,¹ the small extent of the group makes the evaluation of the convergent precarious. In order to estimate the possible error of his parallax, I made the following redetermination, first condensing his data into the four normals in Table XXXVI, chosen to give the most favorable determination of the convergent.

TABLE XXXVI
NORMAL PLACES

	α	δ	100μ	\bar{p}	No.	p COMP.		$(O-C) \sin \lambda$	
						Boss	Kapt.	Boss	Kapt.
$\delta > +18^\circ$	$4^h 19^m$	$+20^\circ 5'$	$11''.6$	113.6	9	113.6	116.1	0.0	-1.1
$\delta < +13$	4 27	+ 9.5	11.0	92.8	7	94.0	90.8	-0.5	+0.7
Rest, α small.....	4 11	+15.9	12.2	104.7	13	104.1	104.5	+0.3	+0.1
Rest, α great.....	4 27	+15.3	10.5	104.3	12	106.1	106.6	-0.8	-0.8
Weighted mean..	4 20	+15.6	11.35

¹ *Astronomical Journal*, 26, 31, 1911.

According to Boss the convergent lies at

$$6^{\text{h}}7^{\text{m}}.2 \quad +6^{\circ}56' \quad (130)$$

which indeed satisfies the observations excellently (last column but one); but

$$5^{\text{h}}46^{\text{m}} \quad +8^{\circ}40' \quad (131)$$

satisfies them almost equally well (last column). For the stream-velocity Boss used only three stars, while we now have the nine objects in Table XXXVII. The values $\rho_{\text{corr.}}$ were obtained by

TABLE XXXVII
RADIAL VELOCITIES, HYADES

α	δ	ρ	Sp	$\rho_{\text{corr.}}$	λ	Authority
3 ^h 8.....	+17°	+25.7	F	+25.7	149°	Küstner, <i>A.N.</i> , 198, 409
[4.0.....	+19	+21.6	K	+19.1	151]	" <i>A.N.</i> , 198, 409
4.2.....	+15	+39.6	K	+37.1	155	" <i>Ap.J.</i> , 27, 301
4.3.....	+17	+40.8	K	+38.3	156	" <i>Ap.J.</i> , 27, 301
4.3.....	+22	+40.0	A ₃	+39.0	155	<i>Lick Bull.</i> , 7, 20
4.3.....	+18	+36.4	A	+35.4	156	" " 7, 20
4.4.....	+19	+39.4	K	+36.9	157	Küstner, <i>Ap.J.</i> , 27, 301
4.4.....	+16	+39.4	K	+36.9	157	" <i>A.N.</i> , 198, 409
4.9.....	+21	+42.0	A ₅	+41.0	161	<i>Lick Bull.</i> , 7, 20
Mean 4.3.....	+18	+36.3	155.8	

reducing Küstner's results to those of the Lick observers by applying the systematic corrections found by Küstner himself and then adding the constant corrections found by Campbell (*Lick Bulletin*, 6, 127); λ is the angular distance from the point (131).

The second star was excluded on account of its great divergence. Forming for the others the equations of condition

$$V \cos \lambda = \rho_{\text{corr.}}$$

and solving by least squares,

$$V = 40.0 \text{ km (8 stars).} \quad (132)$$

This and $100v = 100\mu = 11''.35$ (Table XXXVI) give by (51)

$$\pi = 0''.033; \text{ therefore } M = m - 2.41. \quad (133)$$

The maximum at apparent magnitude 5.36 (126) thus entails

$$K = +2.95. \quad (134)$$

The value (125) furnished by the Pleiades is open to the criticism that it rests on (124), determined from only eight B5 stars. We may obtain a result based on a greater number of stars if we adopt the parallax¹ derived on the hypothesis that the motion of the group in space, referred to the center of gravity of the whole system, is parallel to the Milky Way. The parallax thus found is

$$\pi = 0''.018, \text{ therefore } M = m - 3.72. \quad (135)$$

The value (123), first reduced to the Harvard scale by subtracting 0.24 (see footnote to Table XXXV), thus leads to

$$K = 3.74 \quad (23 \text{ A stars}) \quad (136)$$

Collecting results, we thus have, the adopted weights being in parentheses:

	K		h
	First Solution	Second Solution	
Pleiades (125), (136) ..	4.79 (0.5)	3.74 (1)	(123) 0.82
Hyades (129), (134) ...	2.26 (1)	2.95 (1)	(129) 0.88
Mean	3.10	3.34	0.85

The values of K , which in the first solution were extremely divergent, have come very much nearer together. The mean values for both solutions agree well with (121). The value of h is surprisingly close to that of (122). In conclusion, therefore, the Pleiades and the Hyades furnish values of K which leave much to be desired; still, as far as they go, they decidedly confirm the value (121). I finally adopt

$$K = +3.4. \quad (137)$$

27. REMARKS ON THE GEOMETRICAL FORM OF THE LUMINOSITY-CURVES

a) In what precedes we have assumed that the luminosity-curves are error-curves. In how far is this assumption justified? For stars of all spectra together the form was very decisively

¹ *Proceedings Amsterdam Academy of Sciences*, 14, Part 2, 909, 1912.

indicated by the results in *Groningen Publications*, No. 11, though the representation by an error-curve was not there given. This was first done in *Astronomical Journal*, 24, 115, 1904. The representation over a range of 16 magnitudes leaves little to be desired. For spectra of the first and second types separately, luminosity-curves were also given in *Publication*, No. 11. These, too, are well represented by error-curves. But the data available for these investigations established the curve only as far as the maximum, or nearly so; the fainter branch is altogether wanting. What will be the form for the fainter magnitudes?

The question was considered in *Mount Wilson Contribution*, No. 82, p. 43, but could be answered only very imperfectly. Even now the data are scanty, though somewhat improved, especially for types B and A. They lead to the conclusion that for these spectra the whole curve, both ascending and descending branches, can be represented—with some rough approximation at least—by an error-curve.

Take, for instance, the stars of spectrum B₃ for which the whole curve is pretty well covered by observations. In Table XXXIII a comparison is made between the observed numbers (adopted) and the best-fitting error-curve (computed). The divergences O—C for the fainter part of the curve are admittedly large, but they can hardly be qualified as systematic or as greater than can be explained by the scarcity of stars.

In order to obtain more material, I proceeded as follows. The whole curve is covered by observation for:

B ₅ —B ₉	13 stars in the Pleiades
A	26 " " "
A	19 " " Hyades
B ₃	20 " " Nebula-group
	<hr/>
	78

Since, by Table XXXIV, the best-fitting error-curves do not differ excessively in their values of h , I formed the deviations from the separate means for each of these four groups. The question is whether the positive and negative deviations for all these 78 stars

are symmetrical and in accordance with an error-curve. The result is as follows:

Deviations	Observed	Smoothed	Computed	O-C
<- 2.25.....	0.0	1.0	1.0	0.0
-2.25 to -1.75.....	3.0	1.3	2.2	-0.9
-1.75 " -1.25.....	1.0	6.0	5.2	+0.8
-1.25 " -0.75.....	14.0	10.0	9.5	+0.5
-0.75 " -0.25.....	15.0	12.8	13.5	-0.7
-0.25 " +0.25.....	9.5	14.5	15.2	-0.7
+0.25 " +0.75.....	19.0	13.3	13.5	-0.2
+0.75 " +1.25.....	11.5	11.8	9.5	+2.3
+1.25 " +1.75.....	5.0	5.5	5.2	+0.3
+1.75 " +2.25.....	0.0	1.7	2.2	-0.5
>+ 2.25.....	0.0	0.0	1.0	-1.0

The theoretical curve has been computed with $h=0.70$. The last column shows the divergence of the observations (smoothed by taking means of three consecutive values) from the theoretical curve. The conclusion is as before: the representation by an error-curve is fairly satisfactory for the entire curve.

b) But this resemblance of the luminosity-curve to an error-curve must not be taken too literally. With our present data it is impossible to investigate the matter very closely. Still it may be well to call attention to the fact that even now there are signs indicating that the agreement probably is by no means absolute. Thus the representation for the A stars in Table XXX is defective. As compared with an error-curve, there is a decided excess of very large and very small deviations, and the same thing is indicated in other ways. Hertzsprung¹ has derived the absolute magnitudes of 15 stars belonging to the Ursa Major group. Twelve of these have A spectra, and their absolute magnitudes (increased by 5 mags. to reduce to our scale) range from -1.03 to $+2.20$;² they are accordingly, by Table XXXIV, from -4.4 to -1.2 mags. brighter than the average A star.

It is not at all surprising that the deviations are all on the bright side. Hertzsprung confined himself to Bradley stars and has, by this choice, given strong preference to very luminous stars. In

¹ *Astrophysical Journal*, 30, 139, 1909.

² There seems to be a mistake in the parallax computed for β Aurigae. I have used the correct value. Without the correction the range would be -0.53 to $+2.20$.

fact, we may say that such catalogues as those of Bradley and Boss are really catalogues of exceptionally luminous stars. It is just this absence of stars of mean and faint absolute magnitude from our catalogues which makes the determination of the luminosity-curves so difficult. Nevertheless, in the case of the Ursa Major stars, the deviations seem to be somewhat excessive for a curve whose probable error is ± 0.60 mag.¹ There is accordingly a strong indication of an excess of very luminous stars.

Similarly there seems to be an excess of stars which are absolutely very faint. The companion to α Eridani, which shows an A spectrum, has absolute magnitude $+10.3$, which is 6.9 mags. fainter than the average A star. Other examples might also be given. As already remarked at the end of Section 25 we have, among the B8 stars, such an exceptional object as β Orionis, which is 8.2 mags. brighter than the average B8-B9 stars.

With more extensive data it may become necessary to give up the error-curve as the best representation of the frequencies of the absolute magnitudes. The most convenient form for trial will then probably be the sum of two error-curves having the same maximum but different moduli.

c) I have taken much trouble in deriving what in many cases can be considered only as first approximations to the luminosity-curves in the conviction that such curves form one of the most important elements in attempting to learn the structure of the universe. Scarcity of data is responsible for the fact that in former endeavors stars of all spectra have been grouped together or that, at most, a separation into only two types has been attempted. The results of the present paper will illustrate how crude is such a procedure, for it involves the combination of stars having -2.5 as a

¹ More important than the size of the deviations is the conclusion that the Ursa Major group must contain many more members than those we know, the bulk of which are among the fainter stars. Even now I think we can specify a great number of stars that belong to the group. There is, however, a great difficulty in the way of deciding with certainty, owing to the similarity of the elements of this group (vertex 20^h31^m , $-40^\circ2$; velocity, -18.4 km) with those of the second star-stream, especially the A stars of this stream (vertex 19^h12^m , -47° ; velocity, -18.5 km). For instance, the brightest of Hertzsprung's stars, β Aurigae, fits practically as well in the second stream as in the Ursa Major group. If we assign it to the former, its absolute magnitude changes from -1.03 to $+0.49$.

mean absolute magnitude (Bo stars) with others for which the mean magnitude is $+10.3$ (second-type stars). The corresponding ratio of intensities is more than 100,000 to 1.

To secure entirely reliable results we must attack the spectral classes separately. This will be an immense task, for it involves obtaining the numbers, magnitudes, spectra, and proper motions of a suitable fraction of the faint and very faint stars. In fact the best of the results given in the present paper are due to the invaluable data for stars to magnitudes 9.0 or 9.5 placed at my disposal by Professor Pickering and Miss Cannon.

The requirements for a solid foundation in deriving the luminosity-curves are perhaps best seen from Table XXXVIII, which shows the mean absolute magnitudes for stars of given apparent magnitude and proper motion. It was computed with

TABLE XXXVIII
ABSOLUTE MAGNITUDES M

μ	m		
	6.0	9.0	12.0
0.05.....	+1.2	+3.6	+ 5.9
0.10.....	+2.3	+4.6	+ 7.0
0.20.....	+3.4	+5.7	+ 8.1
0.40.....	+4.4	+6.8	+ 9.2
0.80.....	+5.5	+7.9	+10.2

the data for all stars in *Groningen Publications*, No. 8. To all appearances the parallax for any specified magnitude and proper motion does not differ very markedly for stars of different spectra, so that for a rough estimate a single table can be used. The table shows, what of course is evident, that for the absolutely brightest stars we are dependent on the apparently bright stars of small proper motion. As these stars have been observed more completely and accurately than any others, the conditions are favorable for the brighter end of the luminosity-curve. The faint absolute magnitudes, on the contrary, are to be found among the apparently faint stars of large proper motion. The conditions here are nearly as unfavorable as possible. The table indicates, roughly, how far

we may hope to go with the available data and what will be required for further progress.

Since we have found the mean absolute magnitude of the B₅ stars to be +1.6, and of the A stars to be +3.4, whereas for the second-type stars it is +10.3,¹ it is rather more than a guess if we assign the following series of values:

B ₅	stars	at	+ 1.6
A ₅	"	"	+ 3.4
F ₅	"	"	+ 7
G ₅	"	"	+ 10
K ₅	"	"	+ 13
M	"	"	+ 15

Assuming that the luminosity-curves are error-curves and that for a satisfactory determination of their constants we must possess reliable observational data, at least to a magnitude well past the maximum, we see from Table XXXVIII that Boss's *Catalogue* and the *Revised Harvard Photometry* (*Harvard Annals*, 50), which are complete to magnitudes 5.8 and 6.5, respectively, may be considered as perhaps just sufficient for the derivation of the curve for the A₅ stars. For the F stars we cannot expect altogether reliable results until the stars to 9.0 or somewhat beyond can be included. The *Revised Draper Catalogue* will furnish the required data for the spectra. For the large proper motions (which are the more important) we shall probably be able to manage with what is known, especially when the much-longed-for determinations of proper motion, now in preparation at Albany, have been placed in the hands of astronomers. For the G stars even these data will be inadequate, and we shall have for them no thoroughly sound determination until the data for the Selected Areas become available.

For the K and M stars the outlook might seem to be nearly hopeless. But there is one circumstance which probably will bring success along as soon as the definitive luminosity-curve for the G stars is found. It has long been known that the stars of very large proper motion, almost without exception, are of the second type.

¹ This value was obtained by representing the numbers of *Groningen Publications*, No. 11, p. 31, Type II, Solution B, by an error-curve.

We further find that the larger the proper motion and the fainter the apparent magnitude, the more do the K5-M stars begin to predominate. From van Maanen's table of large proper motions (*Mount Wilson Contribution*, No. 96), I find that among the stars fainter than magnitude 6.0, with proper motion $\geq 1''.0$, 60 per cent are of type K5-M. For still fainter stars we must expect that this percentage will increase largely, so that finally a limit of faintness will be reached for which practically all the stars of large proper motion will be K5-M. Not only must this be so theoretically, but there is a strong indication of its confirmation by observed color indices such as those of Seares and Hertzsprung (*Mount Wilson Contributions*, Nos. 81, 100, and 102).

We therefore expect that beyond the magnitude for which we can determine spectra we shall be justified in including with the K5-M stars all swiftly moving objects, instead of limiting ourselves to those for which the spectra have been ascertained. Perhaps we shall have to multiply our numbers by a certain fraction, slightly less than unity, but it seems probable that this fraction will be capable of a sufficiently accurate determination with the aid of the somewhat brighter stars. Consequently, as soon as the data now under observation for the Selected Areas have become available, we shall probably be able to complete the investigation down to the faintest stars observed for the *Durchmusterung* of the Selected Areas, that is, approximately to:

Mag. 16.0	in the Northern Hemisphere	(Harvard plates)
" 18.2	" " "	(Mount Wilson plates)
" 16.8	" Southern "	(Harvard Arequipa plates)

Of course there will remain the necessity of finding all the stars having large proper motions. This, however, will present no real difficulty. A repetition of the plates, even if made at the present moment, would be sufficient for the purpose. If photographed through the glass and superposed on the originals for differential measurement, or if taken as usual and carefully measured with the stereocomparator, the labor of finding all the proper motions exceeding $0''.10$ would not be very trying. We may thus hope to secure the data necessary for a thorough study of the arrangement

in space of all the stars, including even those, thousands of times fainter than the sun, which are barely visible in our largest telescopes when near, and escape all observation^{*} when situated in the remoter regions of space.

28. SUMMARY

1. The bulk of the B stars within the limits (2) form a local group. The great Orion nebula lies within these limits, and I have therefore called this group the *Nebula-group*.

2. Boss's proper motions in declination, for the stars in the region of the sky considered in the present paper, require the corrections (13) of Section 4.

3. Of the stars outside the Nebula-group, those within the limits $l=180^\circ$ to 216° , $b=-30^\circ$ to $+4^\circ$ may form a local group. It seems more probable, however, that they all have the same systematic motion. I have assumed this to be the case. If later investigation should confirm the existence of the local group, relatively little will be changed in the conclusions of the present paper (Section 7).

4. The definitive elements of the motion of the helium stars outside the nebula region are given in (33), Section 8. The vertex nearly coincides with the vertex of the first stream of the non-helium stars.

5. The mean parallaxes for three separate groups of these stars are given in Table IX (Section 7), the average parallax for all being $0''.0081 \pm 0''.0007$.

6. The error in the position of the vertex due to remaining systematic errors in the proper motions cannot exceed 3° or 4° , and the remaining uncertainties in V and π from this source must be negligible (Section 8).

7. The components u and l of the peculiar motion are distributed approximately in accordance with error-curves. The probable amount of these components is:

for region of present paper (outside stars) $r_u = \pm 1.67$ km
for region of *Mount Wilson Contribution*, No. 82 $r_u = \pm 1.0$ km

^{*} I do not mean that we must be idle until the Selected Areas are completed. On the contrary, I believe that even now we can find very useful representations of the most urgently needed portions of the luminosity-curves.

The smallness of these values is probably the most important fact brought to light by the present investigation.

8. The distribution of u being found, we can derive the distribution of v as soon as we know the fraction of the stars $\phi(\pi)$ having any given parallax π ; $\phi(\pi)$ is determined by the condition that the theoretical distribution of v shall agree with the observed distribution (Section 13).

9. $\phi(\pi)$ being known, it becomes easy to determine the mean parallax of all the stars having given values of v and λ . These mean parallaxes have been adopted as the individual parallaxes of stars having the same v and λ . They have been tabulated for $r_u = \pm 2.5$ in Table XXI, for $r_u = 0.0$ in Table XXIV, and the definitive values, corresponding to $r_u = \pm 1.67$, have finally been obtained by interpolation between these tables.

10. The same method applied to τ does not lead to valuable results (Section 16).

11. The parallax of any star more than 24° from the antivertex, derived in the manner described, is, roughly speaking, $\pm 0''.0021$. This includes the effect of the observational errors in the proper motions (Section 15).

12. The parallax of the Nebula-group, $\pi = 0''.0054 \pm 0''.0009$, has been found by a consideration of the luminosity-curves (Section 18).

13. To the parallaxes various tests have been applied. In general they are strongly confirmatory, both for the outside stars and for the Nebula-group (Sections 17 and 19).

14. The systematic motion of the Nebula-group can differ but little from the motion of the outside stars (Section 20).

15. Luminosity-curves have been determined for many subdivisions of the B stars. Further, a curve was also derived for the A stars. All the results are summarized in Table XXXIV.

16. The frequency-curve of the absolute magnitudes (luminosity-curve) has in former publications been found to coincide nearly with a normal error-curve; in consequence of a lack of data, in particular for the faint stars, the agreement could not be established beyond a point near the maximum. In the present paper occur several cases for which the descending branch could also be included more or less completely. From all available material it appears that the

TABLE XXXIX
B STARS, GAL. LONG. 150° TO 210°

Boss No.	Sp.	HARV. MAG.	1900		GAL. LONG.	GAL. LAT.	CORRECTION (13) APPLIED		VERT. 17 ^h 48 ^m , +0°		100 τ	100 ν	100 r	$\rho - 4.3$	ρ comp.	UNIT 0.0001		
			α	δ			100 μ	ρ	ρ comp.	λ						$\pi_{2.5}$	$\pi_{4.4}$	M

a) LAT. 0° TO +30°, 100 $\mu \cong 1:7$																		
6135....	B5	4.06	6 ^h 23 ^m	+20.3	160°	+5°	2.2	201°	198°	149°	-0.1	+2.2	0.23	km	km	88	90	89
5550....	B3	4.92	6 6	+10.2	161	+8	2.4	195	195	154	+1.0	+2.3	0.44	94	101	96
1558....	B0	5.28	6 10	+10.2	161	+8	2.1	193	192	153	+1.0	+1.8	0.36	+17.8	+17.8	94	101	96
1781....	B8	3.68	6 44	+10.3	166	+7	2.3	230	210	151	-1.0	+1.5	0.52	+8.5W	+16.5	88	84	85
1788....	B8	3.88	6 51	+9.5	172	+13	2.7	240	220	155	-1.2	+2.4	0.60	+28.7W	+18.1	101	113	105
1944....	B8	3.09	7 22	+8.5	177	+13	2.5	220	233	158	-0.5	+2.5	0.18	103	240	189
2071....	B8	3.11	7 47	+8.5	186	+18	2.5	250	249	148	-0.3	+2.2	0.30	94	90	96
2350....	B3	4.32	8 44	+3.8	191	+25	2.2	205	253	136	+1.0	+2.4	0.40	79	70	76
2385....	B8	3.59	8 44	+3.6	198	+25	2.9	220	280	130	+1.0	+2.4	0.40	82	77	80
1935....	B8	3.50	7 20	-10.0	199	+1	3.4	260	285	156	+1.4	+3.8	0.74	-11.0W	+18.3	118	149	130
2431....	B8	3.50	9 2	-8.2	207	+25	3.0	245	265	132	+1.4	+3.8	0.74	87	82	85
2492....	B0	3.54	9 12	-8.3	207	+25	4.0	274	265	130	-0.6	+4.0	0.46	+6.7W	+12.8	115	114	115
2642....	B3	3.53	7 40	-24.4	208	+1	3.9	301	294	148	-0.5	+3.9	0.70	121	147	130

b) LAT. 0° TO -30°, 100 $\mu \cong 1:7$																		
075....	B8	5.15	4 0	+0.8	151	-27	4.2	170	138	149	-2.8	+3.1	0.72	105	122	111
1315....	B3	3.81	5 22	+21.0	151	-6	1.7	140	168	148	+0.8	+3.5	0.32	122	166	170
1315....	B3	3.28	5 30	+21.0	151	-6	1.7	140	168	148	+0.8	+3.5	0.32	106	119	110
081....	B3	3.32	4 10	+8.6	152	-28	3.2	126	127	149	+0.1	+3.2	0.43	+18.0W	+16.0	112	132	119
1007....	B0	5.37	4 44	+12.0	153	-16	2.4	165	158	152	-0.1	+3.4	0.44	+13.7W	+17.7	95	103	98
1165....	B8	5.74	4 50	+14.0	153	-16	2.4	145	147	152	-0.1	+2.8	0.71	+4.7W	+17.7	105	103	98
1375....	B3	3.00	4 32	+21.1	153	-13	2.8	172	172	149	-0.1	+2.8	0.18	105	112	104
1203....	B2	4.65	4 50	+15.3	153	-13	4.1	152	152	153	0.0	+2.1	0.28	128	168	141
1404....	B8	5.89	5 30	+19.7	157	-13	2.2	177	180	151	+0.1	+2.2	0.34	90	94	91
1504....	B8	5.17	5 36	+16.5	158	0	2.8	172	185	150	+0.6	+2.7	0.40	90	110	103
1306....	B3	4.87	5 36	+16.5	159	-6	3.1	155	173	154	+1.0	+2.0	0.40	+27.7	+18.0	108	129	115
1048....	B0	6.01	4 23	+1.2	161	-30	3.5	143	117	157	-1.4	+3.2	0.48	124	158	135
1448....	B0	5.41	4 47	+14.4	161	-6	2.8	172	178	157	-0.3	+2.8	0.38	116	141	123
1455....	B0	5.57	4 47	+14.1	161	-5	1.7	230	170	157	-1.3	+3.1	0.35	72	64	69
1084....	B5	5.32	4 32	+0.8	162	-28	1.8	220	110	158	-1.8	+0.3	0.45	+18.3W	+18.5	51	20	44
1438....	B0	4.92	5 44	+12.6	162	-6	2.9	207	178	158	-1.4	+2.5	0.44	110	133	118

TABLE XXXIX—Continued

1525...	B3	4.40	6 2	+14.8	162	-2	3.7	166	188	155	+1.4	+	3.4	0.20	121	153	112	0.00
1530...	B2	4.35	6 9	+14.2	164	-1	3.5	165	101	156	+1.5	+	3.2	0.36	121	152	131	-0.06
1537...	B2	5.81	6 9	+13.9	164	-1	4.1	74	103	155	+3.6	+	2.0	0.66	121	131	30...	-1.23...
1572...	B0	1.70	5 20	+6.3	165	-15	2.0	200	158	162	-1.3	+	1.5	0.16	+19.3	85	90	87	-3.60
1577...	B0	5.36	6 10	+12.6	165	-1	1.7	135	103	156	+1.4	+	0.9	0.36	+13.7	68	57	64	-0.61
1581...	B3	4.32	5 25	+5.9	166	-14	3.8	162	162	163	-0.1	+	3.8	0.36	141	187	150	+0.28
1613...	B5	4.18	4 41	+3.4	168	-28	1.8	112	110	162	-0.1	+	1.8	0.2	92	104	96	+0.01
1618...	B5	4.18	4 41	+3.4	168	-28	1.8	112	110	162	-0.1	+	1.8	0.2	83	83	83	+0.67
1632...	B2	4.73	5 20	+1.8	168	-17	2.1	209	154	166	-1.7	+	1.2	0.62	90	80	90	+0.07
1633...	B2	5.30	5 28	+1.7	174	-17	2.4	182	134	170	-1.8	+	1.6	0.74	71	60	67	-0.25
1637...	B1	5.32	5 32	+6.1	178	-23	1.6	259	80	169	-0.0	+	1.6	0.2	+13.7	57	57	57	-0.54
1638...	B1	5.32	5 32	+6.1	178	-23	1.6	259	80	169	-0.0	+	1.6	0.2	71	60	67	-0.25
1639...	B1	5.75	5 33	+6.0	178	-18	2.4	272	80	172	-1.0	+	2.4	1.06	+19.6	93	65	84	+0.37
1640...	B3	5.02	6 29	+1.1	180	-3	4.7	226	290	166	+0.5	+	2.7	0.66	158	200	172	+1.20
1641...	B8	4.54	5 8	-12.0	181	-20	3.2	108	55	168	-2.6	+	1.9	0.39	95	105	98	-0.50
1642...	B8	6.88	6 22	-4.3	181	-6	2.0	84	252	168	+0.4	+	2.0	0.57	71	40	61	+0.81
1643...	B8	4.46	5 9	-13.1	182	-26	2.0	246	52	169	+0.5	+	1.9	0.45	73	40	62	-1.58
1644...	B3	4.73	6 24	-7.0	184	-7	3.7	278	279	169	+0.1	+	3.7	0.30	+19.6	143	167	151	+0.62
1645...	B2	5.22	6 17	-11.7	188	-11	2.7	251	319	170	+2.5	+	3.7	0.84	70	78	70	+0.02
1646...	B2	5.40	7 2	-11.1	192	0	2.2	236	274	161	+1.4	+	1.7	0.52	80	97	92	+0.10
1647...	B5	5.29	6 44	-15.0	194	0	1.6	195	200	164	+1.6	+	0.1	0.80	65	47	59	-0.86
1648...	B3	5.20	6 52	-22.8	202	-8	2.7	306	308	159	+0.1	+	2.7	1.00	115	144	125	+0.74
1649...	B3	5.06	6 55	-25.3	203	-8	2.8	321	311	157	-0.5	+	2.7	0.58	113	138	131	+1.08
1650...	B3	5.80	6 57	-25.1	204	-8	3.6	318	310	156	-0.5	+	2.3	0.78	120	167	142	+1.56
1651...	B3	5.75	7 3	-23.7	204	0	2.3	310	306	156	-0.2	+	2.3	0.96	103	118	108	+0.92
1652...	B3	5.76	7 6	-25.1	205	-6	2.2	280	307	156	+1.0	+	2.0	0.59	93	100	95	+0.65
1653...	B8	5.09	7 23	-22.7	206	-2	2.2	43	208	153	-2.1	+	0.6	1.14	50	28	43	-1.14
1654...	B	5.04	6 1	-32.2	206	-22	13.1	359	353	150	-1.3	+	13.0	1.66	300*	400*	333	+3.25
1655...	B5	2.75	5 30	-34.1	207	-28	2.7	172	7	154	-0.7	+	2.6	0.24	125	160	137	+0.57
1656...	B5	4.89	5 40	-33.8	207	-25	3.6	2	0	155	-0.1	+	3.6	0.74	+20.7	100	100	96	+0.71
1657...	B5	5.80	6 25	-32.3	208	-17	2.2	321	339	155	+0.7	+	2.1	0.70	119	150	120	+0.03
1658...	B8	4.48	6 24	-32.5	209	-18	3.7	311	339	155	+1.7	+	3.3	0.44	+20.7(1)	103	103	98	-0.54
1659...	B8	4.50	7 35	-26.6	210	-1	3.4	341	300	148	-2.2	+	2.6	0.61	+18.7	98	108	101	+0.54
1660...	B3	4.02	7 44	-29.0	210	-5	2.7	304	307	149	+0.1	+	2.7	0.54	115	137	122	+0.05
1661...	B3	4.02	7 35	-26.6	210	-1	3.6	307	300	148	-0.4	+	3.6	0.60	112	137	120	+1.00
1662...	B3	5.00	6 17	-34.1	210	-10	3.3	358	344	153	-0.8	+	3.2	0.74	107	127	114	+0.60
1663...	B5	5.31	7 11	-30.5	211	-8	3.2	300	314	151	+0.8	+	3.1	0.85	118	145	127	+0.59
1664...	B8	4.55	7 31	-28.1	211	-3	7.1	255	303	149	+5.3	+	4.8	0.60	92	95	93	+0.64
1665...	B5	5.07	6 55	-34.0	213	-12	3.7	327	325	150	-0.1	+	3.7	0.68	102	113	106	+1.11
1666...	B3	5.80	7 25	-31.2	213	-5	2.8	278	310	148	+1.5	+	2.4	0.76	124	149	132	+0.22
1667...	B3	5.98	7 23	-33.9	215	-7	4.1	270	313	146	+2.8	+	3.0	0.71	124	149	132	+0.22
1668...	B8	4.62	7 34	-34.7	216	-6	4.2	302	310	144	+0.6	+	4.2	0.71	124	149	132	+0.22

*By extrapolation of the value for $\frac{v}{\mu}$, guiding ourselves by the way in which $\frac{v}{\mu}$ changes in *Gron. Publ.* No. 8, p. 31.

TABLE XXXIX—Continued

[illegible]

d) SUPPLEMENTARY LIST

[illegible]

* Mean of two components of bright Hy excluded.

agreement with an error-curve extends, with some rough approximation, over the whole curve, though there are signs indicating some excess of extreme luminosities in the case of the B8-B9, and A stars.

NOTATION

The notation used is the same as in *Mount Wilson Contribution*, No. 82, except that t has been substituted for v (Section 11). Only what is necessary for the understanding of Table XXXIX is repeated here.

v, τ	components of the angular proper motion μ along the great circle toward the antivertex, and at right angles thereto, respectively.
100 r	100 times probable error in the proper motion of any co-ordinate (Section 10).
$p-4.3$	the observed radial velocity corrected by the constant amount -4.3 km, in accordance with <i>Mount Wilson Contribution</i> , No. 82, p. 28.
W	observed at Mount Wilson.
:	observation uncertain.
orb.	spectroscopic binary, radial velocity of system obtained from determination of orbit.
est.	spectroscopic binary, radial velocity of system estimated.
(1)	only one observation.
$\pi_{2.5}$	parallax on the supposition that $r_u = \pm 2.5$
$\pi_{0.0}$	" " " " $r_u = 0.0$
π adopted	" " " " $r_u = \pm 1.67$
n	when the parallax is marked with n the star has been assumed to belong to the Nebula-group.

GRONINGEN

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} See (47) and Section
12, Remark 2.